Asymptotic linearity and Hadamard differentiability

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Abstract

Motivated by the study of solutions of second order nonlinear elliptic equations in the usual Sobolev spaces $W^{2,p}(\mathbb{R}^N)$ for $1 \leq p < \infty$, we present a variant of the standard notion of asymptotic linearity of a mapping $M : X \to Y$ acting between Banach spaces X and Y. For the associated inversion, $M^*(u) = \|u\|^2 M(u/\|u\|^2)$, this new property is equivalent to Hadamard differentiability at 0. New results about bifurcation for Hadamard differentiable problems then lead to conclusions about asymptotic bifurcation for nonlinear elliptic equations on $\mathbb{R}^N$.