Workshop on Nonlinear Elliptic PDE's and Applications

Granada, December 15-16, 2011

Abstracts

Quasilinear elliptic problems with natural growth in the gradient and superlinear nonlinearities

David Arcoya, Universidad de Granada

For a continuous function $g \geq 0$ on $(0, +\infty)$ (which may be singular at zero), we confront a quasilinear elliptic differential operator with natural growth in ∇u with a superlinear nonlinearity. The model case is the study of the problem

$$\begin{gathered} -\Delta u + g(u)|\nabla u|^2 = \lambda(1+u)^p \quad \text{in } \Omega \\ u = 0 \text{ on } \partial\Omega. \end{gathered}$$

where f is given either by $f(x,u) = \lambda u^p + f_0(x)$ ($f_0 \gneqq 0$), or by $f(x,u) = \lambda(1+u)^p$. The range of values of the parameter λ for which the associated homogeneous Dirichlet boundary value problem admits positive solutions depends on the behavior of g and on the exponent p. This is joint work with José Carmona and Pedro J. Martínez-Aparicio.

Solutions with mixed positive and negative spikes for some semilinear elliptic problems.

Teresa D'Aprile, Università di Roma II (Tor Vergata)

We consider the following stationary nonlinear Schrödinger equation

$$-\varepsilon^2 \Delta v + v = f(v) \quad \text{in } \Omega, \tag{1}$$

where Ω is a smooth and bounded domain of $\mathbb{R}^N$, ε is a small positive parameter, f is a superlinear, subcritical and odd nonlinearity. No geometrical or topological assumption on Ω is required. We discuss some recent results concerning concentration phenomena for sign-changing solutions in the semiclassical limit $\varepsilon \to 0^+$ ([1]-[2]). More specifically, we explain the construction of solutions consisting of mixed positive and negative interior peaks for the Dirichlet and the Neumann problems associated to (1). The peaks approach separate points of Ω and their location depends on the geometry of the domain.

References

[1] T. D'Aprile. *Solutions with many mixed positive and negative interior spikes for a semilinear Neumann problem*, Calc. Var. Partial Differential Equations **41** (2011), 435–454.

[2] T. D'Aprile, A. Pistoia. *Nodal solutions for some singularly perturbed Dirichlet problems*, Trans. Amer. Math. Soc. **363** (2011), 3601–3620.

On the pullback equation for differential forms

Bernard Dacorogna, EPFL, Lausanne

An important question in geometry and analysis is to know when two $k-$forms f and g are equivalent. The problem is therefore to find a map φ such that

$$\varphi^* (g) = f.$$

We will mostly discuss the symplectic case $k = 2$ and the case of volume forms $k = n$. We will give some results on the more difficult case where $3 \le k \le n-2$, the case $k = n-1$ will also be considered.

[1] Bandyopadhyay S. and Dacorogna B., On the pullback equation $\varphi^* (g) = f$, *Ann. Inst. Henri Poincaré, Analyse Non Linéaire*, **26** (2009), 1717-1741.

[2] Bandyopadhyay S., Dacorogna B. and Kneuss O., The pullback equation for degenerate forms, *Disc. Cont. Dyn. Syst. Series A*, **27** (2010), 657-691.

[3] Dacorogna B. and Kneuss O., Divisibility in Grassmann algebra, to appear in *Linear and Multilinear Algebra.*

[4] Csato G., Dacorogna B. and Kneuss O., *The pullback equation for differential forms*, to appear with Birkhaüser.

Large solutions at large

Louis Dupaigne, Université de Picardie Jules Verne

The title of this talk answers simultaneously three questions, pertaining to three distinct mathematical subjects: conformal geometry, branching stochastic processes, and nonlinear elliptic PDEs. This motivates us to study so-called boundary blow-up solutions (large solutions, for short), introduced independently by J.B. Keller and R. Osserman. That is, solutions to the equation

$$\Delta u = f(u), \qquad \text{in } \Omega,$$

that converge to $+\infty$ as x approaches the boundary $\partial\Omega$.

Instability of the Yamabe equation

Pierpaolo Esposito

In this talk, I will describe a work in progress (jointly with A. Pistoia and J. Vetois) about the existence of blowing-up solutions for linear perturbations of the Yamabe equation. The Schoen conjecture –concerning the compactness of Yamabe metrics– is known to hold also when perturbing the linear part from below.

Instead, we show that perturbations of the linear part from above immediately give rise to non-compact sequence of solutions. The blow-up points can be simple or even non-simple (accumulation of bubbles and bubbles' tower).

On the classification of entire solutions of quasilinear elliptic equations

Alberto Farina, Université de Picardie Jules Verne

We study entire solutions of quasilinear elliptic equations (possibly very degenerate and/or very singular) of the form

$$\operatorname{div} \mathcal{A}(x, u, Du) = \mathcal{B}(x, u, Du), \qquad x \in \mathbf{R}^N \qquad (*)$$

In particular, under some new and very general assumptions on $\mathcal{A}$ and $\mathcal{B}$, we prove that equation $(*)$ has the following *strong Bernstein-Liouville property:*

$$u \equiv 0 \quad \textit{is the } \underline{\textit{only}} \textit{ entire solution of equation } (*).$$

No conditions are placed on the behavior of the solutions at infinity, neither on their sign. We also investigate the behavior of solutions when the parameters of the problem do not allow the *Bernstein-Liouville property*, showing thus, that the afore-mentioned results are essentially sharp. This is a joint work with James Serrin.

On some models of growth

Ireneo Peral, Universidad Autónoma de Madrid

We will review some result for the Kadar-Parisi-Zhang model of roughening of surfaces and we will propose some mathematical models arising in the theory of epitaxial growth of crystal. We focalize the study on a stationary problem which presents special analytical difficulties, it is in fact a borderline case. We study the existence of solutions. The central model in this work is given by the following elliptic equation,

$$\Delta^2 u = \det\left(D^2 u\right) + \lambda f, \quad x \in \Omega \subset \mathbb{R}^2.$$

The framework to study the problem deeply depends on the boundary conditions.

Some related results and problems will be given.

REFERENCES

B. Abdellaoui, A. Dall'Aglio, I.Peral, *Some Remarks on Elliptic Problems with Critical Growth in the Gradient*, J. Diff. Equations, Vol 222, No 1 (2006) 21-62.

B. Abdellaoui, A. Dall'Aglio, I.Peral, *Regularity and nonuniqueness results for parabolic problems arising in some physical models, having natural growth in the gradient.* J. Math. Pures Appl. 90 (2008) 242-269

Carlos Escudero, Ireneo Peral, *The stationary problem associated to a variational model of epitaxial growth.* preprint.

On the best Lipschitz extension problem for a discrete distance and the discrete ∞-Laplacian

Julio Rossi, Universidad de Alicante

This talk is concerned with the best Lipschitz extension problem for a discrete distance that counts the number of steps. We relate this absolutely minimizing Lipschitz extension with a discrete ∞-Laplacian problem, which arise as the dynamic programming formula for the value function of some ε-tug-of-war games. As in the classical case, we obtain the absolutely minimizing Lipschitz extension of a datum f by taking the limit as $p \to \infty$ in a nonlocal p–Laplacian problem.

Joint work with J Mazon and J Toledo.

TBA.

Boyan Sirakov, Université de Paris X

A quasilinear elliptic Dirichlet problem having two positive solutions

Charles A. Stuart, EPFL, Lausanne

For a class of second order quasilinear elliptic equations we establish the existence of two non-negative weak solutions of the Dirichlet problem on a bounded domain, Ω. Solutions of the boundary value problem are critical points of C^1-functional on $H_0^1(\Omega)$. One solution is a local minimum and the other is of mountain pass type.

Theoretical study of an elliptic system with a chemotaxis term and non-linear boundary condition

Antonio Suárez, Universidad de Sevilla

In this talk, we present some theoretical results of a non-linear elliptic system including chemotaxis term and non-linear boundary condition. This system is related with the angiogenesis process, crucial step in the tumor's growth. Specifically, this problem models the effect of an anti-angiogenic therapy based on an agent that binds specific receptors of the endothelial cells.

“Local” and “global” variational eigenvalues of the p-Laplacian and the Fredholm alternative

Peter Takáč, Universität Rostock

We begin this lecture with a discussion of the famous Ljusternik-Schnirelmann characterization of *some* eigenvalues of nonlinear elliptic problems (by a minimax formula) which has a *global* variational character. Then we show that for some homogeneous quasilinear elliptic eigenvalue problems there are variational eigenvalues *other* than those of the Ljusternik-Schnirelmann-type. In contrast, these eigenvalues have a *local* variational character. Such phenomenon does not occur in typical linear elliptic eigenvalue problems, thanks to the Courant-Fischer theorem which is the linear analogue and predecessor of the Ljusternik-Schnirelmann theory. Finally, we present some existence and multiplicity results for the Fredholm alternative at the first (smallest) eigenvalue of the Dirichlet p-Laplacian. We will show some hints for proofs which combine variational and topological methods with linearization procedures.