Trigonometría

Relaciones Trigonométricas Fundamentales

- $\operatorname{sen} a \cdot \operatorname{cosec} a = 1$
- $\cos a \cdot \sec a = 1$
- $\operatorname{tg} a = \sin a / \cos a$
- $\operatorname{tg} a \cdot \operatorname{cotg} a = 1$
- $\operatorname{sen}^2 a + \cos^2 a = 1$
- $1 + \operatorname{tg}^2 a = \sec^2 a$
- $1 + \operatorname{cotg}^2 a = \operatorname{cosec}^2 a$
- $\operatorname{sen}(a \pm b) = \operatorname{sen} a \cos b \pm \cos a \operatorname{sen} b$
- $\cos(a \pm b) = \cos a \cos b \mp \operatorname{sen} a \operatorname{sen} b$
- $\operatorname{tg}(a \pm b) = (\operatorname{tg} a \pm \operatorname{tg} b)/(1 \mp \operatorname{tg} a \operatorname{tg} b)$
- $\operatorname{sen}(a + b) + \operatorname{sen}(a - b) = 2 \operatorname{sen} a \cos b$
- $\operatorname{sen}(a + b) - \operatorname{sen}(a - b) = 2 \cos a \operatorname{sen} b$
- $\cos(a - b) + \cos(a + b) = 2 \cos a \cos b$
- $\operatorname{sen}(a - b) - \cos(a + b) = 2 \operatorname{sen} a \operatorname{sen} b$

$$A = \frac{a+b}{2} \qquad \text{y} \qquad B = \frac{a-b}{2}$$

- $1 + \cos(a + b) = 2 \cos A$
- $1 - \cos(a - b) = 2 \operatorname{sen} A$
- $\operatorname{sen} 2a = 2 \operatorname{sen} a \cos a$
- $\cos 2a = \cos^2 a - \operatorname{sen}^2 a = 2\cos^2 a - 1 = 1 - 2\operatorname{sen}^2 a$
- $\operatorname{sen} 2a = (2 \operatorname{tg} a)/(1 + \operatorname{tg}^2 a)$
- $\cos 2a = (1 - \operatorname{tg}^2 a)/(1 + \operatorname{tg}^2 a)$
- $\operatorname{sen}^2 a = 1/(1 + \operatorname{tg}^2 a)$
- $\cos^2 a = \operatorname{tg}^2 a/(1 + \operatorname{tg}^2 a)$
- $\operatorname{tg} 2a = (2 \operatorname{tg} a)/(1 - \operatorname{tg}^2 a)$
- $\operatorname{sen} 3a = 3 \operatorname{sen} a - 4 \operatorname{sen}^3 a$
- $\cos 3a = 4 \cos^3 a - 3 \cos a$
- $\operatorname{sen} na = 2 \operatorname{sen}\big((n-1)a\big) \cos a - \operatorname{sen}\big((n-2)a\big)$
- $\cos na = 2 \cos\big((n-1)a\big) \cos a - \cos\big((n-2)a\big)$
- $\operatorname{sen} \frac{a}{2} = \pm\sqrt{\frac{1-\cos a}{2}} \begin{cases} + & , \text{ si } \frac{a}{2} \text{ en cuadrantes 1º ó 2º} \\ - & , \text{ si } \frac{a}{2} \text{ en cuadrantes 3º ó 4º} \end{cases}$

- $\cos\frac{a}{2} = \pm\sqrt{\frac{1+\cos a}{2}} \begin{cases} + & \text{, si } \frac{a}{2} \text{ en cuadrantes } 1^o \text{ ó } 4^o \\ - & \text{, si } \frac{a}{2} \text{ en cuadrantes } 2^o \text{ ó } 3^o \end{cases}$
- $\operatorname{tg}\frac{a}{2} = \pm\sqrt{\frac{1-\cos a}{1+\cos a}} = \frac{1-\cos a}{\operatorname{sen} a} = \frac{\operatorname{sen} a}{1+\cos a} \begin{cases} + & \text{, si } \frac{a}{2} \text{ en cuadrantes } 1^o \text{ ó } 3^o \\ - & \text{, si } \frac{a}{2} \text{ en cuadrantes } 2^o \text{ ó } 4^o \end{cases}$
- $\operatorname{sen} a + \operatorname{sen} b = 2\operatorname{sen} A \cos B$
- $\operatorname{sen} a - \operatorname{sen} b = 2\cos A \operatorname{sen} B$
- $\cos a + \operatorname{sen} b = 2\operatorname{sen}(\frac{\pi}{4} - B)\cos(\frac{\pi}{4} - A)$
- $\cos a - \operatorname{sen} b = 2\operatorname{sen}(\frac{\pi}{4} - A)\cos(\frac{\pi}{4} - B)$
- $\cos a + \cos b = 2\cos A \cos B$
- $\cos a - \cos b = -2\operatorname{sen} A \operatorname{sen} B$
- $\operatorname{tg} a + \operatorname{tg} b = \frac{\operatorname{sen}(2A)}{\cos a \cos b}$
- $\operatorname{tg} a - \operatorname{tg} b = \frac{\operatorname{sen}(2B)}{\cos a \cos b}$
- $\operatorname{cotg} a + \operatorname{tg} b = \frac{\cos(2B)}{\operatorname{sen} a \cos b}$
- $\operatorname{cotg} a - \operatorname{tg} b = \frac{\cos(2A)}{\operatorname{sen} a \cos b}$
- $\operatorname{cotg} a + \operatorname{cotg} b = \frac{\operatorname{sen}(2A)}{\operatorname{sen} a \operatorname{sen} b}$
- $\operatorname{cotg} a - \operatorname{cotg} b = \frac{\operatorname{sen}(2B)}{\operatorname{sen} a \operatorname{sen} b}$

Ángulos negativos, complementarios, suplementarios, etc.

	$-a$	$90 \pm a$	$180 \pm a$	$270 \pm a$	$n360 \pm a$
sen	$-\operatorname{sen} a$	$+\cos a$	$\mp\operatorname{sen} a$	$-\cos a$	$\pm\operatorname{sen} a$
cos	$+\cos a$	$\mp\operatorname{sen} a$	$-\cos a$	$\pm\operatorname{sen} a$	$+\cos a$
tg	$-\operatorname{tg} a$	$\mp\operatorname{ctg} a$	$\pm\operatorname{tg} a$	$\mp\operatorname{cotg} a$	$\pm\operatorname{tg} a$
cosec	$-\operatorname{cotg} a$	$\mp\operatorname{tg} a$	$\pm\operatorname{cotg} a$	$\operatorname{tg} a$	$\operatorname{cotg} a$
sec	$+\sec a$	$\operatorname{cosec} a$	$-\sec a$	$\operatorname{cosec} a$	$+\sec a$
cotg	$-\operatorname{cotg} a$	$\operatorname{tg} a$	$\pm\operatorname{cotg} a$	$\mp\operatorname{tg} a$	$\pm\operatorname{cotg} a$

Equivalencia en radianes y valor de las funciones de ángulos usuales

	rad	sen	cos	tg	cosec	sec	cotg
0	0	0	1	0	∞	1	∞
30	$\pi/6$	$1/2$	$\sqrt{3}/2$	$1/\sqrt{3}$	2	$2/\sqrt{3}$	$\sqrt{3}$
45	$\pi/4$	$\sqrt{2}/2$	$\sqrt{2}/2$	1	$2/\sqrt{2}$	$2/\sqrt{2}$	1
60	$\pi/3$	$\sqrt{3}/2$	$1/2$	$\sqrt{3}$	$2/\sqrt{3}$	2	$1/\sqrt{3}$
90	$\pi/2$	1	0	∞	1	∞	0
180	π	0	-1	0	∞	-1	0
270	$3\pi/2$	-1	0	∞	-1	∞	0
360	2π	0	1	0	∞	1	∞