# Influence of the recovery method on the optimum sensors for spectral imaging of skylight

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## ABSTRACT

The profit of low-cost, multispectral imaging systems in estimating spectral power distributions has been widely studied. There are various mathematical methods available (PCA, Wiener's estimation method, spline interpolation, MDST, among others) which permit the accurate reconstruction of a spectrum from the response of a small set of sensors. One important issue in this task is the influence of noise, its propagation through mathematical transformations and how the selection of the sensors of the multispectral system, combined with the spectral estimation algorithm chosen, may reduce its influence. We report here on four different spectral recovery methods that reconstruct skylight spectra from the responses of three Gaussian sensors (the spectral profile of which is a Gaussian curve). The sensors are searched for using a simulated annealing algorithm, and they are optimized so that they give the best possible spectral and colorimetric reconstructions, even in the presence of noise. We show here how the accuracy of the reconstructions is influenced by the recovery method chosen.

Keywords: Spectral imaging systems; spectral estimation methods; illuminant estimation; noise; simulated annealing.

# **1. INTRODUCTION**

The use of multispectral colour systems for registering high-information colour images is becoming increasingly common. No one nowadays, for example, would attempt to analyze naked-eye atmospheric phenomena such as rainbows, halos, glories, or coronas without using some kind of instrument. Daylight and skylight spectra are normally measured with spectroradiometers, which are complex and expensive instruments that provide only one spectrum per measurement. Thus when one measures skylight, the illumination arrives from either a small or large angular subtense of the sky, depending on the instrument's field of view. Because researchers could benefit from high-resolution, angular maps of skylight's spectral power distribution (SPD) we need to measure many skylight spectra simultaneously across the sky dome. Multispectral imaging systems let us do so, for instance, in an attempt to replace classical spectroradiometers at the atmosphere observation stations with lighter and cheaper multispectral systems, which can provide complete spectral information of the skylight impinging upon them at each pixel of the image. This would lead the way to calculating important parameters used in the study of the particle size and composition of aerosols or their concentration in the air (the Angstrom exponent or the optical depth for example<sup>1</sup>) or an automatic software-based method for cloud detection<sup>2</sup>.

In recent years the development and design of multispectral color-image-acquisition devices has received much attention in colour science. By extending our past research on sky color<sup>3-6</sup>, we offer here a theoretical optimum design of a 3-channel multispectral system that can recover the SPD of the skylight incident upon it. Our optimum multispectral system must estimate the spectral skylight radiance at each pixel of the image based on the response of the system's channels. This is a classic inverse problem that requires a mathematical estimation algorithm.

Various estimation algorithms exist which permit us to obtain approximate skylight spectra from the response of sensors. These methods are commonly based on an *a priori* knowledge of the kind of spectra we want to recover. For example, performing a principal component analysis (PCA)<sup>7</sup> on a set of previously registered spectral measurements (called 'training spectra') provides a set of vectors which can be linearly combined to obtain the spectral estimation. The weights in this linear combination are chosen to minimize the mean square error of the estimation in the space of

spectral curves over all the training spectra. Three of the four methods we study here are based on PCA: the Maloney-Wandell method<sup>8</sup> (which has been widely used by other authors<sup>9,10</sup>), the Imai-Berns method<sup>11</sup> and the Shi-Healey method<sup>12</sup>. Another way of including *a priori* spectral knowledge is to develop a Wiener pseudoinverse<sup>13</sup> (also called direct-pseudoinverse<sup>14,15</sup>), where the sensors' responses to the known training spectra are used later to construct a matrix that provides unknown spectra from their measured responses.

The accuracy of the spectral reconstructions obtained from the responses of an optimum set of Gaussian sensors included in an hypothetical multispectral imaging system depends on several interrelated factors: the spectral sensitivity of its sensors, the type and number of sensors, the estimation method chosen, the number and quality of the training spectra and the noise that always affects any electronic device. To include all these factors in an exhaustive search is a highly demanding computational task. Our alternative approach greatly reduces computing time using as it does a simulated annealing algorithm<sup>16</sup> that minimizes a cost function. To this end we propose a single cost function that evaluates the quality of our recovered skylight spectra: the colorimetric and spectral combined metric or CSCM function<sup>10</sup>, which has proved to be a good metric for evaluating spectral and colorimetric differences between skylight spectra<sup>17</sup>.

In this work we study the influence of the estimation method chosen upon the shape of the optimum sensors found. We also show the lowest number of training spectra that should be used in each method. Finally, we make a brief comparative study of the accuracy of each of the four spectral estimation methods mentioned as a function of the noise present in the system and the number of basis vectors used in the linear combinations.

### 2. ESTIMATING SKYLIGHT SPECTRA FROM BROADBAND SENSOR DATA

In a previous paper<sup>10</sup> we simulated the spectral response of CCD camera sensors assuming this response to be linear<sup>7,9,18</sup>. If we make this assumption for our multispectral imaging system, we can model its sensors' response using

$$\rho = R^{t}E \tag{1}$$

where we have sampled the visible spectrum at N different wavelengths and assumed vector notation for the resulting magnitudes (hence the integrals become linear combinations or, equivalently, matrix products). In eq. (1)  $\rho$  is the column vector representing k sensor responses (k = 3 here), E is the illuminant spectrum (skylight in our case), represented by an N x 1 column vector, and R is an N x k matrix containing the spectral sensitivities of the k sensors at N sampled wavelengths (superscript t denotes its transpose). The mission here is to recover the skylight spectra, E, from the measured sensor responses,  $\rho$ . This task cannot be achieved simply by pseudoinverting matrix R, because the projection of a spectrum, E, in the  $\rho$  responses space leads to a significant loss of information. Different estimation methods try to solve this problem.

Any real imaging system is of course affected by noise<sup>7,10</sup>, a fact not explicitly accounted for in eq.(1). Yet noise can be represented as an additive term that changes the ideal noise-free sensors' responses to

$$\rho_{noise} = \rho + \sigma \tag{2}$$

where  $\sigma$  is a *k-row* vector of uncorrelated components that affect each sensor separately. There are various sources of noise<sup>19</sup>, with *thermal noise* being the most common. Another noise source in electronic systems is *shot noise*, the source of which is current fluctuations in semiconductor devices due to the quantum character of electrons. *Flicker noise* is also common, and this varies inversely with camera exposure time. Finally, *quantization noise* is present in every analog-to-digital (A/D) conversion and is the loss of least-significant digits when quantizing scene radiances to a given number of bits (i.e. to a fixed number of discrete levels).

In this study, we simulate thermal and shot noise as random normally distributed noise with standard deviations of 1%, 3%, or 5% of the maximum sensor response (these noise levels correspond to signal-to-noise ratios (SNR) of

40dB, 30dB, and 26dB respectively). Quantization noise is represented as that due to A/D conversion at a resolution of 12 bits altough we demonstrated in a previous work<sup>10</sup> the slight influence of the quantization noise, using at least 8 bits, once the optimum sensors have been selected. This slightly degraded performance closely approximates the behavior of a real multispectral imaging system.

As we said in the introduction, it is common to make use of *a priori* knowledge of the spectra we want to recover, PCA being a widely used strategy. PCA consists of obtaining a set of vectors (called principal components or eigenvectors) which can then be used to express a given spectrum as a linear combination in terms of

$$E = V\varepsilon \tag{3}$$

where *V* is an *N* x *n* matrix containing the first *n* eigenvectors used for reconstructing *N* wavelengths (*n* is always less than or equal to *N* and is usually chosen to equal *k*, the number of sensors, which tends to give the best results<sup>11,20</sup>). Vector  $\varepsilon$  is a *n*-rowed vector that contains the coefficients of the linear combination. The first three methods we are about to present make use of this linear approximation for the spectra.

#### 2.1 Maloney-Wandell method:

This method<sup>8</sup> simply makes the substitution of eq. (3) in eq. (1) to obtain

$$\rho = R' V \varepsilon = \Lambda \varepsilon \tag{4}$$

Then  $\Lambda$  is a *k* x *n* matrix that directly transforms the coefficients  $\varepsilon$  into the sensor responses  $\rho$ . By calculating  $\Lambda$ 's pseudoinverse (denoted by superscript <sup>+</sup>) we obtain the coefficients for the linear estimate of the spectrum from the camera sensors' responses, and we can then recover the skylight spectrum as

$$E_R = V\Lambda^+ \rho \tag{5}$$

In this method the *a priori* information provided by the training spectra is included in matrix V (which contains the eigenvectors or principal components), which also appears in  $\Lambda$  as can be seen in eq.(4). We also notice that it is necessary to measure the spectral sensitivities, R, of the k sensors to obtain matrix  $\Lambda$ .

#### 2.2 Imai-Berns method:

Imai and Berns developed a method<sup>11</sup> for reflectance recovery based directly on a relation<sup>14</sup> between the sensor responses,  $\rho$ , and coefficients  $\varepsilon$ , which now includes a column in  $\rho_{ts}$  and  $\varepsilon_{ts}$  for each of the *m* training spectra (we will use *m* = 20, 156 and 1567 in this study, as we will explain later). Thus

$$\varepsilon_{ts} = G\rho_{ts} \tag{6}$$

In this new equation the system matrix *G* is an *n* x k matrix which is formally similar to  $\Lambda^+$  in eq.(5), but now is determined empirically by a least-squares analysis from the training-vector measurements. Hence it is not necessary to measure the spectral sensitivities, *R*, of the camera to use this method with real sensor-response measurements<sup>14</sup>. We can estimate the matrix, *G*, as a result of a least-squares analysis, by pseudoinverting the *k* x *m* matrix,  $\rho_{ts}$ :

$$G = \varepsilon_{ts} \rho_{ts}^+ \tag{7}$$

In our case, the recovered skylight spectrum is simply calculated in this method from the measured sensor responses,  $\rho,$  as

$$E_R = VG\rho \tag{8}$$

Here, the information provided by the training spectra is included in *V* and in *G*. Cheung *et.al.*<sup>20</sup> use this same method by pseudoinverting  $\varepsilon_{ts}$  in eq.(6) instead of  $\rho_{ts}$ . The resulting matrix is the pseudoinverse of our *G*, which should be pseudoinverted again in eq.(8). Although theoretically the recovered spectra,  $E_R$ , should be the same, the additional pseudoinverse operation introduces artificial noise in the algorithm, and the results are poorer.

#### 2.3 Shi-Healey method:

Shi and Healey<sup>12</sup> presented a new method which permits them to use higher dimensional models for reflectance and illuminant spectra in eq.(3). Although the methods of Maloney-Wandell and Imai-Berns can be used with more eigenvectors than sensors (n > k), this does not lead to the best results -as we will see later- because a model with n > kdoes not determine a unique mapping between  $\rho$  and  $\varepsilon$ , since *E* vectors having different  $\varepsilon$  values can generate the same  $\rho$  vector<sup>12</sup> as a result of a loss of information when registering an n > k linear model with only *k* parameters (the sensors' responses). We call the set, *S<sub>E</sub>*, of vectors, *E*, generated when varying the *n* coefficients  $\varepsilon$  (more than the number of sensors *k*) and having the same responses  $\rho$ . In order to associate a unique *E<sub>R</sub>* recovered illuminant vector with a  $\rho$  measurement vector we can select a single vector, *E*<sup>\*</sup>, from the set, *S<sub>E</sub>*, with the constraint of requiring that *E*<sup>\*</sup> be the vector in *S<sub>E</sub>* that minimizes the mean-square error calculated over all the training spectra. In other words, we will choose *E*<sup>\*</sup> from a given  $\rho$  as that vector which is most similar to a training spectrum among those vectors of *S<sub>E</sub>* that are consistent with both the linear model and the sensor vector,  $\rho$ .

Since we have three sensors (k = 3) in this preliminary study of our planned multispectral system, given a dimensionality *n* for the linear model, we separate the contributions of the last three principal components (denoted by subscript 2) and the remaining *n*-3 first principal components (subscript 1) in eq.(4) and thus

$$\rho = R^{t} \left( V_{1} \varepsilon_{1} + V_{2} \varepsilon_{2} \right) \tag{9}$$

where  $V_1$  contains the eigenvectors 1,...,*n*-3 and  $V_2$  contains the eigenvectors *n*-2,...,*n*. The vectors  $\varepsilon_1$  and  $\varepsilon_2$  contain the corresponding coefficients for the linear estimation. In fact Cheung *et.al.*<sup>20</sup> use this method, but they separate the first three basis vectors from the remaining n - 3 last basis vectors, which leads to poorer reconstructions in all the cases we studied in section 4, and thus we have omitted here the results obtained with this variation of the Shi-Healey method. From eq.(9) we can resolve  $\varepsilon_2$  in terms of  $\varepsilon_1$  according to

$$\varepsilon_2 = \left( R^t V_2 \right)^{-1} \left( \rho - R^t V_1 \varepsilon_1 \right) \tag{10}$$

and substituting in eq.(3)

$$E = V_1 \varepsilon_1 + V_2 \left( R^t V_2 \right)^{-1} \left( \rho - R^t V_1 \varepsilon_1 \right)$$
<sup>(11)</sup>

From this equation, we can construct an N x m matrix,  $E^*$ , of column vectors of  $S_E$  that minimizes the mean-square-error over all the training spectra. This can be achieved by using pseudoinversion in eq.(11) to find  $\varepsilon_1$ , thus

$$E^{*} = V_{1}\varepsilon_{1}^{*} + V_{2}\left(R^{t}V_{2}\right)^{-1}\left(\hat{\rho} - R^{t}V_{1}\varepsilon_{1}^{*}\right)$$
(12)

where the (*n*-3) x m matrix  $\varepsilon_1^*$  is given by the following relation ( $\hat{\rho}$  is a 3xm matrix which contains the sensors' responses,  $\rho$ , to the measured spectra, *E*, repeated in its *m* columns):

$$\varepsilon_{1}^{*} = \left(V_{1} - V_{2}\left(R^{t}V_{2}\right)^{-1}R^{t}V_{1}\right)^{+}\left(E_{ts} - V_{2}\left(R^{t}V_{2}\right)^{-1}\hat{\rho}\right)$$
(13)

 $E_{ts}$  is an  $N \times m$  matrix containing one training spectrum per column. We have constructed an  $N \times m$  matrix,  $E^*$ , of estimated spectra from the sensor responses,  $\rho$ , of a measured spectrum, E. Each column of  $E^*$  is related to each column of  $E_{ts}$ , which contains the training spectra. If we calculate the distance between each column of  $E^*$  to each column of  $E_{ts}$ , we can choose the estimated spectrum,  $E_R$ , as that column of  $E^*$  for which that distance is minimum:

$$E_R = E_i^* \tag{14}$$

*i* selects the column of  $E^*$  for which the distance  $\left\|E_i^* - E_{ts_i}\right\|$  is minimum. The most important disadvantage of this

method is that for every given vector response,  $\rho$ , we have to calculate *m* estimated spectra for  $E^*$  and choose the minimum of *m* distances. If *m* is large, the algorithm is extremely slow. We also need to measure accurately the spectral sensitivities, *R*, of the camera. This method will be used here with n = 4 and 5 basis vectors, since if we use just 3 basis vectors the matrix  $V_1$  would be zero and eq.(12) would be exactly eq.(5) for the Maloney-Wandell method for 3 sensors and 3 basis vectors ( $\Lambda$  would be a square 3x3 matrix).

### 2.4 Wiener estimation method

The Wiener estimation method<sup>13-15</sup> is formally similar to Imai and Berns' method although now it relates the sensor responses,  $\rho$ , directly with the spectral estimations,  $E_R$ , using a matrix, W, on the way, and so

$$E = W\rho \tag{15}$$

We can estimate W using a least-squares approach, calculating the pseudoinverse of  $\rho$  for the training spectra:

$$W = E_{ts} \rho_{ts}^{+} \tag{16}$$

The recovered spectrum will be given by a relation identical to eq.(15) using the previously constructed W matrix with eq.(16). It is not necessary to measure the spectral sensitivities of the camera. The information of the training spectra is included in W, but we do not construct a linear basis for representing the spectra; we just try to build a "robust" matrix, W, and introduce the sensors' responses into eq.(15) to obtain the spectral estimations.

# **3. SEARCHING FOR OPTIMUM SENSORS**

Various methods have been proposed for selecting the optimum filters or sensors for a multispectral imaging system<sup>7,9,15,18</sup>. No consensus of opinion exists, however, about what "optimum" means in such a system. For us, one set of sensors is clearly better than another if its reconstructed skylight spectra are more accurate when tested by some metric. The key question is what metric to use. For our problem, essentially two kinds of metric exist: colorimetric and spectral<sup>17,21</sup>. Colorimetric metrics such as those proposed by the CIE (CIELUV, CIELAB, CIE94, or CIEDE2000) approximate color differences observed by the human eye. Spectral metrics are those which measure the distance between two spectral curves, such as RMSE or GFC<sup>4</sup> (which uses Schwartz's inequality, a widely accepted<sup>17,21,22</sup> index of similarity between two spectra). These metrics distinguish between metamers but do not consider human visual system, such as weighted RMSE (WRMSE) with the diagonal of Cohen's matrix R<sup>21</sup>. Viggiano proposed a spectral comparison index (SCI)<sup>22</sup>, the properties of which have been studied by others<sup>17,21,22</sup>. Another metric widely used in solar radiation measurements is the percentage of the integrated irradiance error<sup>23</sup> [IIE(%)] across the visible spectrum.

Some authors<sup>9</sup> have searched for optimum sensors using only one of the metrics described above. Because their results depend on the metric used they are not particularly useful in selecting sensors for our planned multispectral system. Imai *et al.* suggest that "mononumerosis" should be avoided when evaluating the quality of spectral matches<sup>21</sup>. By this term they mean that *several* metrics should be used to assess color reconstruction from both colorimetric and

spectral standpoints. Day<sup>15</sup> used thresholds for RMSE and CIEDE2000 metrics when searching for optimum sensors; Hernández-Andrés *et al.*<sup>3</sup> used GFC, CIELUV and IIE(%) in a similar way.

We must use a single cost function when developing a simulated annealing algorithm. This approach may seem to contradict the recommendations of Imai *et al.*<sup>21</sup> Yet it does not, because we actually use a simple single-cost function or metric that combines several metrics at once. We use GFC as a spectral metric, CIELAB  $\Delta E_{ab}^*$  as a colorimetric cost function, and IIE(%) as a metric for comparing the spectral curves of natural illuminants. In principle, this metric should approach zero for near-perfect matches (in contrast with GFC, which tends to unity for perfect matches) and give approximately the same weight to the GFC, CIELAB  $\Delta E_{ab}^*$ , and IIE(%) metrics. Our combined CSCM metric has proved to be a good metric for comparing skylight spectra, and is calculated<sup>10,17</sup> by

$$CSCM = Ln(1+1000(1-GFC)) + \Delta E_{ab}^{*} + IIE(\%)$$
(17)

Now that Eq. (17) defines our optimum sensor quantitatively, we can turn to developing a search algorithm. Whenever possible one should make an exhaustive search to find the optimum sensors for any multispectral system. Yet such a search can demand excessive computer time because the number of possible sensor combinations can be enormous. We perform our study using 3 sensors that are Gaussian functions of wavelength<sup>9,10,18</sup>. We vary sensor peak sensitivities from 380-780 nm in 5nm steps, a spectral resolution suitable for both colorimetry and radiometry. We also vary the sensors' FWHM (full width at half maximum) from 10-300 nm in 5-nm steps and the peak value of the sensors from 0.5 to 1 in 0.1 steps. Finally, we perform linear spectral recoveries using 3, 4, and 5 eigenvectors. To appreciate the computational burden involved, note that ~  $10^{13}$  different sets must each be evaluated to find the optimum set for a 3-sensor system, a search that requires several days on existing personal computers. Faced with such daunting computational challenges, we turn to *simulated annealing* algorithms<sup>10,16</sup>, which have been widely used as search algorithms in physics and greatly speed up the finding of optimum solutions to a system with many different sets of sensors. If we slightly relax our requirements for recovery accuracy, an annealing algorithm can give a nearly optimum solution after testing only ~  $10^5$  sets of sensors<sup>10</sup>.

### **4. RESULTS**

In this section we compare the accuracy obtained with each of the four methods presented for various sizes of the set of training spectra, for various noise situations and different numbers of basis vectors (in those methods where PCA is necessarily performed).

For this study we used a previously obtained set of 1567 skylight spectral measurements taken in Granada<sup>4</sup>, Spain (37°10'N, 3 36'W, elevation 680m) at many different solar elevations; each spectrum ranged from 380-780 nm in 5-nm steps. We used this complete set as a test set in all the recovery experiments. This same set served us as a training set for the system in three different situations: we chose the complete set of 1567 measurements, and two subsets of 156 and 20 measurements respectively to compare the efficiency of the algorithms related to the number of training spectra. The subsets of 156 and 20 skylight spectra were selected from the original set of 1567 spectra by selecting one of each ten and seventy-eight measurements respectively (the original set was arranged according to the date in which the measurements were taken). This selection was made just once, and proved to assure a high variety in the shape of the training spectra in all the cases.

#### 4.1 Optimum sensors

We found that the shape (peak locations and FWHM) of the optimum sensors was almost always the same in every noise situation and for every number of basis vectors used within each recovery method. This behavior is desirable for developing a practical multispectral system. As other authors have noted<sup>9,10,18</sup>, sensor sensitivity curves tend to sharpen when the noise is high (i.e., low SNR; except for the Imai-Berns method). In the three methods using PCA, there was no significant difference in the shape of the optimum sensors when varying the number of basis vectors used. We also found that the optimum sensors obtained with the Maloney-Wandell method and the Wiener method are

very similar in every situation. The optimum sensors obtained with the Shi-Healey method are very peculiar, they seem to be equally spaced in the visible and are very narrow-band.

### 4.2 Accuracy in the reconstructions

In this section we present the results obtained when recovering our 1567 skylight spectra of the original set with the optimum sensors found using each of the four methods presented as a function of the size of the training set of spectra *m*. We show the results using the CSCM metric explained in the previous section. Table 1 sets out the results of this study for the Maloney-Wandell method. We show in each row the results obtained using a different number of basis vectors. We separate into columns the three cases of simulated noise (corresponding to SNRs of 40dB, 30dB and 26dB) and the number of training spectra used in every noise situation. Each table cell contains mean values and their corresponding standard deviations (SD) for the CSCM metric. Analogous tables are presented for the other recovery methods: Table 2 for the Imai-Berns method, Table 3 for the Wiener method (in this case no basis vector is needed) and Table 4 for the Shi-Healey method.

	40dB				30dB		26dB		
	<i>m</i> = 1567	<i>m</i> = 156	m = 20	<i>m</i> = 1567	<i>m</i> = 156	m = 20	<i>m</i> = 1567	<i>m</i> = 156	m = 20
3 basis	2.0±1.1	2.1±1.1	$2.0{\pm}1.2$	4.1±2.1	4.1±2.1	4.1±2.1	6.1±3.3	6.3±3.3	6.1±3.4
vectors									
4 basis	2.2±1.2	$2.2 \pm 1.2$	2.2±1.2	4.3±2.0	$4.2\pm2.0$	4.2±2.0	6.0±3.0	6.3±3.0	$6.2 \pm 3.2$
vectors									
5 basis	2.3±1.3	$2.5 \pm 1.4$	2.4±1.3	4.6±2.1	4.7±2.1	4.6±2.1	6.4±3.0	6.7±3.0	6.6±3.2
vectors									

Table 1. Mean values ± standard deviations for the Maloney-Wandell method and the CSCM metric

	40dB				30dB		26dB		
	<i>m</i> = 1567	<i>m</i> = 156	m = 20	<i>m</i> = 1567	<i>m</i> = 156	m = 20	<i>m</i> = 1567	<i>m</i> = 156	m = 20
3 basis	2.0±1.1	2.1±1.1	2.1±1.2	3.6±1.8	3.6±1.8	3.7±1.8	5.2±2.9	$5.3 \pm 2.8$	5.5±2.7
vectors									
4 basis	2.0±1.1	$2.0\pm0.9$	2.1±1.2	3.9±1.8	$4.0{\pm}2.1$	3.8±1.8	$5.9 \pm 2.8$	$5.9 \pm 3.0$	$5.4 \pm 2.9$
vectors									
5 basis	2.0±1.1	2.0±0.9	2.0±1.0	3.9±1.8	$4.0 \pm 1.8$	4.1±1.8	5.9±3.0	6.0±3.0	6.0±2.9
vectors									

**Table 2.** Mean values ± standard deviations for the Imai-Berns method and the CSCM metric

	40dB			30dB		26dB			
<i>m</i> = 1567	<i>m</i> = 156	m = 20	<i>m</i> = 1567	<i>m</i> = 156	m = 20	<i>m</i> = 1567	<i>m</i> = 156	m = 20	
2.0±1.1	2.1±1.1	2.1±1.2	3.9±2.1	3.9±1.8	3.9±1.9	6.0±3.2	$6.2 \pm 3.1$	6.0±3.2	
<b>Table 3.</b> Mean values + standard deviations for the Wiener method and the CSCM metric									

	40dB			30dB			26dB		
	<i>m</i> = 1567	<i>m</i> = 156	<i>m</i> = 20	<i>m</i> = 1567	<i>m</i> = 156	<i>m</i> = 20	<i>m</i> = 1567	<i>m</i> = 156	m = 20
4 basis	1.3±0.8	1.6±1.0	2.0±1.4	2.5±1.3	2.7±1.4	3.3±1.9	3.9±2.1	4.0±2.2	4.8±2.7
vectors									
5 basis	1.3±0.7	$1.5 \pm 1.0$	2.0±1.1	2.4±1.3	2.6±1.6	3.0±1.7	3.8±2.1	3.8±2.1	$4.0\pm2.1$
vectors									

Table 4. Mean values  $\pm$  standard deviations for the Shi-Healey method and the CSCM metric

We can see how for the Maloney-Wandell and the Imai-Berns methods the best choice is to use 3 basis vectors when recovering skylight spectra from three-sensor responses, as other authors have found before<sup>8-12,20</sup>. The Maloney-Wandell and Wiener methods achieve similar results in all cases whilst the Imai-Berns method improves a little as noise increases. There is a noticeable improvement in the quality of the recoveries for high noise if we use the complete training set of spectra. The Shi-Healey method has clearly turned out to be the best method for recovering spectra from

three-sensor responses, and we can see how increasing its dimensionality results in lower values for the CSCM metric<sup>12,20</sup>.

### **5. CONCLUSIONS**

We have shown the similarities and differences between the optimum sensors found for recovering skylight spectra from noisy broadband sensor data with four different methods. The Maloney-Wandell and Wiener methods, although very different mathematically, present a similar behaviour in the shape of the optimum sensors and the quality of the recoveries. The Imai-Berns method improves a little upon these two methods when noise is high. The Shi-Healey method has proved to be the best for the task of recovering skylight spectra from the responses of three sensors, although it is extremely slow when using a large training set.

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