The Dirichlet problem for a singular elliptic equation arising in the level set formulation of the inverse mean curvature flow

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Resumen

In this lecture we consider the Dirichlet problem associated with a nonlinear singular elliptic equation arising in the level set formulation of the inverse mean curvature flow; namely,

$$-\mathrm{div}\left(\frac{Du}{|Du|}\right) + |Du| = f\,.$$

We introduce a suitable concept of weak solution, for which we prove existence and uniqueness of the homogeneous Dirichlet problem in a bounded open set of $\mathbb{R}^N$, in the case $0 \leq f \in L^q(\Omega)$, $q > N$.

The *inverse mean curvature flow* is a one-parameter family of hypersurfaces $\{\Gamma_t\}_{t\geq 0} \subset \mathbb{R}^N$ $(N \geq 2)$ whose normal velocity $V_n(t)$ at each time t equals the inverse of its mean curvature $H(t)$. If we let $\Gamma_t := F(\Gamma_0, t)$, then the parametric description of the inverse mean curvature flow is to find $F : \Gamma_0 \times [0,T] \to \mathbb{R}^N$ such that

$$\frac{\partial F}{\partial t} = \frac{\nu}{H}, \qquad t \geq 0, \tag{1}$$

where ν denotes the unit outward normal to Γ_t.

Huisken and Ilmanen in [1] propose a level set formulation for the inverse mean curvature flow (1), and by means of their existence and uniqueness results about this formulation of the inverse mean curvature flow, they then give a proof of a conjecture from the black hole theory known as *Penrose Inequality.* The level set formulation propose in [1] is the following. Assume that the flow is giving by level set of a function $u : \mathbb{R}^N \to \mathbb{R}$ via

$$\Gamma_t = \partial E_t, \quad E_t := \{x \in \mathbb{R}^N \ : \ u(x) < t\}.$$

Wherever u is smooth with $\nabla u \neq 0$, equation (1) is equivalent to

$$\mathrm{div}\left(\frac{\nabla u}{|\nabla u|}\right) = |\nabla u|.$$

Our approach is different to that introduced in [1] and follows the ideas developed to study the Dirichlet problem for the total variation flow (see [2]).
Sección en el CEDYA 2011: EDP

Bibliography

[1] G. Huisken and T. Ilmanen, *The Inverse Mean Curvature Flow and the Riemannian Penrose Inequality*, J. Differential Geom. **59** (2001) 353–438.

[2] F. Andreu, C. Ballester, V. Caselles and J.M. Mazón, *The Dirichlet Problem for the Total Variational Flow*, J. Funct. Anal. **180** (2001), 347–403.