An approximation problem in computing electoral volatility

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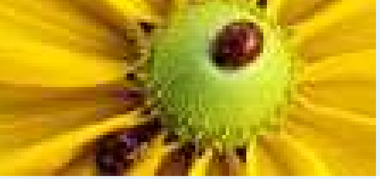
Reference: Applied Mathematics and Computation,

Vol. 192, No. 2, pp. 299–310, 2007. doi: 10.1016/j.amc.2007.03.032



Introduction

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- Though the volatility formula offers no mathematical complexity, its application to real data may present several problems.

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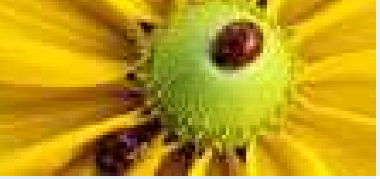

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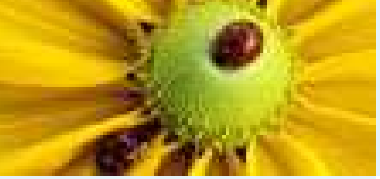

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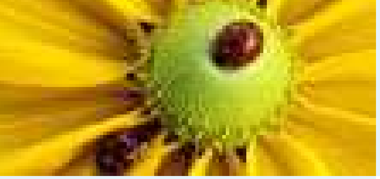
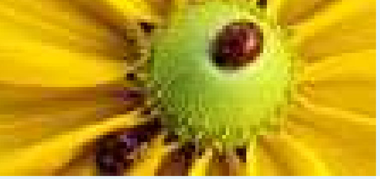


Approximation of volatility



Computing volatility in practice

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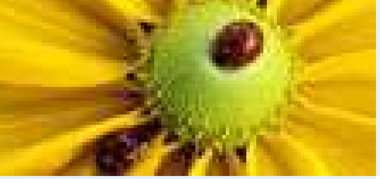

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$$\{(p_i, q_i) \; : \; i \in \mathcal{I}\} \; ,$$

where p_i and q_i stand for the electoral strengths (votes, seats, etc.) of the (party) unit $i \in \mathcal{I}$ in both elections, respectively.

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To measure the electoral change, the PV measure (Pedersen, 1979) could be computed by

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Though the PV formula is very simple, the following computational problems can arise.

Computational problems for $\mathbb{V}$

Problem 1: when the parties in both elections are not the same.

Computational problems for V

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Solution: Guidelines proposed in Bartolini and Mair (1990, Appendix 1, pp. 311–312)

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Some situations:

- When the party category *Others* appears in the electoral data and no information is given for parties included in this category (Bartolini and Mair, 1990, Appendix 1).
- When details on parties with non significant strength are omitted to maintain memory capabilities under minimum requirements in databases.
- When the researcher is only interested in *relevant parties* (Sartori, 1976, p. 122) and, thus, those non–relevant parties are not considered for the BM rules

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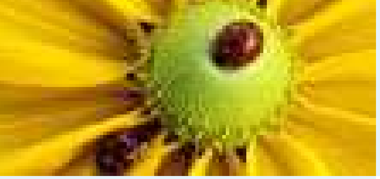
Consequence: V cannot be evaluated.



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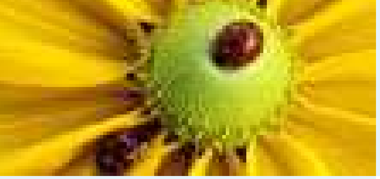
In this paper the approximation of V in such a framework is analyzed.

Approximations of volatility and errors



Problem setting

Assume that only M pairs of strengths are the available data, for any $M \in \mathcal{I}$, which are given by

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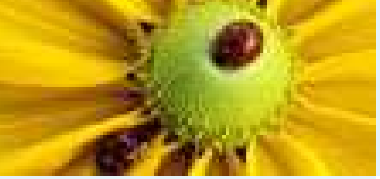
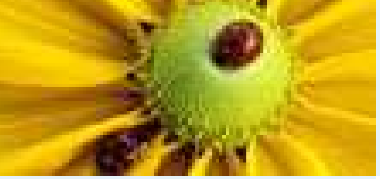
$\mathbb{V}$ is then unknown.



Study of approximations of V

A naive approximation of V is given by

$$\mathrm{V}[M] \;=\; \frac{1}{2}\sum_{k=1}^{M}\left|p_k - q_k\right|, \qquad (2)$$



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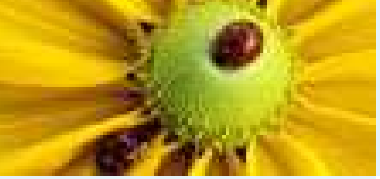

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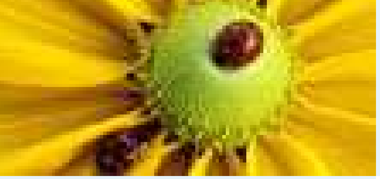

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$$\mathrm{P}[M] = 1 - \sum_{k=1}^{M} p_k \quad \text{and}$$

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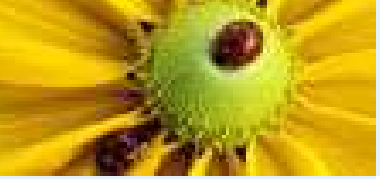

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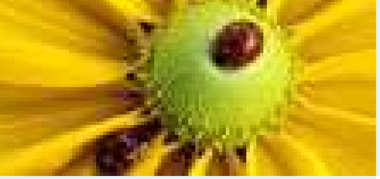
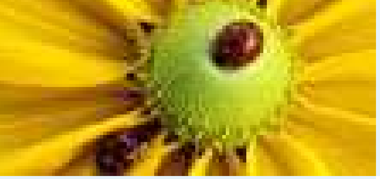
Remark: $\mathrm{VL}[M]$ is the BM approximation. It was used in Bartolini and Mair (1990).



A new approach

The study of the error for the BM approximation suggests an alternative approach given by

$$\mathrm{V}_\theta[M] \;=\; \theta\ \mathrm{VU}[M] + (1-\theta)\ \mathrm{VL}[M]\,, \quad \forall\, M \in \mathcal{I} \text{ and } \forall\, \theta \in [0,1]\,. \tag{4}$$



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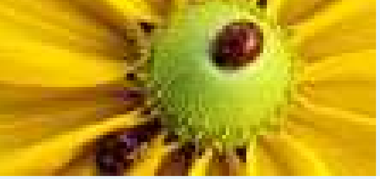

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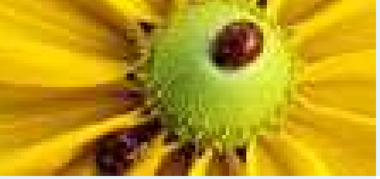
$$1/2 = arg \min_{\theta\in[0,1]} \mathrm{EC}_\theta[M]$$



Upper error bounds

In these paper, it is derived that

$$\mathrm{EA}[M] \;\leq\; \mathrm{EC}_\theta[M] \;\leq\; \mathrm{EBM}[M]\,, \quad \forall\,\theta \in [0,1] \text{ and } \forall\, M \in \mathcal{I},$$

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Moreover, it is proved that

$$\mathrm{EA}[M] = \frac{1}{2}\,\mathrm{EBM}[M]\ .$$

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