The full lepton flavor of little Higgs

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in collaboration with

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- Little Higgs: *Littlest* Higgs with T parity (LHT)
- **New** sources of flavor mixing
- Lepton flavor changing processes
- Conclusions

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+ work in progress

Little Higgs | Solution to hierarchy problem

Hierarchy problem: the Higgs mass should be of order v (electroweak scale) but it receives quadratic loop corrections of the order of the theory cutoff (Planck scale?)

Naturalness $\Rightarrow$ New Physics at the TeV scale

[In SUSY: cancellations of quadratic Higgs mass corrections provided by superpartners]

In **LH models** the Higgs is a pseudo-Goldstone boson of an approximate global symmetry broken at f (TeV scale) [Arkani-Hamed, Cohen, Georgi '01]

(i) Product group

the SM $SU(2)_L$ group from the diagonal breaking of two or more gauge groups

e.g.: *Littlest* Higgs [Arkani-Hamed, Cohen, Katz, Nelson '02]

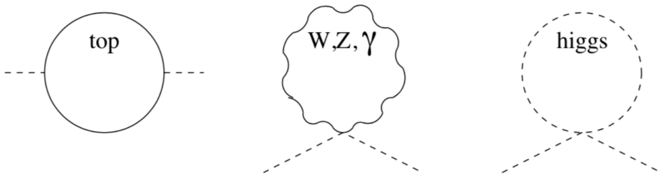
(ii) Simple group

the SM $SU(2)_L$ group from the breaking of a larger group into an $SU(2)$ subgroup

e.g.: *Simplest* Little Higgs ($SU(3)$ simple group) [Kaplan, Schmaltz '03]

Little Higgs **Effective theory**

– The low energy *dof* described by a nonlinear sigma model, an effective theory valid below a cutoff $\Lambda \sim 4\pi f$ (order of 10 TeV) since then the loop corrections are



$$\Delta M_h^2 \sim \{y_t^2, g^2, \lambda^2\} \frac{\Lambda^2}{16\pi^2} \lesssim (1 \text{ TeV})^2$$

Ultraviolet completion (unknown) is required only for physics above Λ

– The global symmetry explicitly broken by gauge and Yukawa interactions, giving the Higgs a mass and non-derivative interactions, preserving the cancellation of *one-loop* quadratic corrections thanks to **collective symmetry breaking**:

the symmetry is broken only when ***two or more*** *couplings are non-vanishing*

LH introduce extra scalars, fermions and gauge bosons: new flavor mixing sources

$\Rightarrow$ Enhanced predictions for lepton flavor changing processes

$$h \to \bar{\ell}_1 \ell_2, \quad Z \to \bar{\ell}_1 \ell_2, \quad \ell_1 \to \ell_2 \gamma, \quad \mu \text{N} \to \text{eN}, \quad \ell \to \ell_1 \ell_2 \bar{\ell}_3$$

Littlest Higgs

[Arkani-Hamed, Cohen, Katz, Nelson '02]

(1) $SU(5) \to SO(5)$ by $\Sigma_0 = \begin{pmatrix} \mathbf{0}_{2\times 2} & 0 & \mathbf{1}_{2\times 2} \\ 0 & 1 & 0 \\ \mathbf{1}_{2\times 2} & 0 & \mathbf{0}_{2\times 2} \end{pmatrix}$, $\Sigma(x) = e^{i\Pi/f}\Sigma_0 e^{i\Pi^T/f} = e^{2i\Pi/f}\Sigma_0$

where $\Pi(x) = \phi^a(x)X^a$ and X^a are the $24 - 10 =$ 14 broken generators $\Rightarrow$ 14 GB

$G \equiv SU(5) \supset [SU(2)\times U(1)]_1 \times [SU(2)\times U(1)]_2$ **(gauge)** $\xrightarrow{\langle\Sigma\rangle=\Sigma_0} SU(2)_L \times U(1)_Y$

[unbroken]: $Q_1^a + Q_2^a,\ Y_1 + Y_2 \ \Rightarrow$ 4 gauge bosons (γ, Z, W^+, W^-) remain massless

[broken]: $Q_1^a - Q_2^a,\ Y_1 - Y_2 \ \Rightarrow$ 4 gauge bosons (A_H, Z_H, W_H^+, W_H^-) get masses of order f

4 WBGB $(\eta, \omega^0, \omega^+, \omega^-)$ eaten by (A_H, Z_H, W_H^+, W_H^-)

10 GB: $\underbrace{H \text{ (complex } SU(2) \text{ doublet)}}$, Φ (complex $SU(2)$ triplet, PGB)

(2) EWSB: $SU(2)_L \times U(1)_Y \xrightarrow{\langle H\rangle} U(1)_{\rm QED} \Rightarrow H = \sqrt{2}\begin{pmatrix} -i\frac{\pi^+}{\sqrt{2}} \\ \frac{v+h+i\pi^0}{2} \end{pmatrix}$

3 WBGB (π^0, π^+, π^-) eaten by (Z, W^+, W^-)

1 PGB: h

Littlest Higgs | Goldstones and gauge generators

The matrix of the 14 Goldstone Bosons:

$$\Pi = \begin{pmatrix} -\frac{\omega^0}{2} - \frac{\eta}{\sqrt{20}} & -\frac{\omega^+}{\sqrt{2}} & -\mathrm{i}\frac{\pi^+}{\sqrt{2}} & -\mathrm{i}\Phi^{++} & -\mathrm{i}\frac{\Phi^+}{\sqrt{2}} \\ -\frac{\omega^-}{\sqrt{2}} & \frac{\omega^0}{2} - \frac{\eta}{\sqrt{20}} & \frac{v+h+\mathrm{i}\pi^0}{2} & -\mathrm{i}\frac{\Phi^+}{\sqrt{2}} & \frac{-\mathrm{i}\Phi^0 + \Phi^P}{\sqrt{2}} \\ \mathrm{i}\frac{\pi^-}{\sqrt{2}} & \frac{v+h-\mathrm{i}\pi^0}{2} & \sqrt{\frac{4}{5}}\eta & -\mathrm{i}\frac{\pi^+}{\sqrt{2}} & \frac{v+h+\mathrm{i}\pi^0}{2} \\ \mathrm{i}\Phi^{--} & \mathrm{i}\frac{\Phi^-}{\sqrt{2}} & \mathrm{i}\frac{\pi^-}{\sqrt{2}} & -\frac{\omega^0}{2} - \frac{\eta}{\sqrt{20}} & -\frac{\omega^-}{\sqrt{2}} \\ \mathrm{i}\frac{\Phi^-}{\sqrt{2}} & \frac{\mathrm{i}\Phi^0 + \Phi^P}{\sqrt{2}} & \frac{v+h-\mathrm{i}\pi^0}{2} & -\frac{\omega^+}{\sqrt{2}} & \frac{\omega^0}{2} - \frac{\eta}{\sqrt{20}} \end{pmatrix}$$

Littlest Higgs | Gauge generators

Generators of the gauge subgroup $[SU(2) \times U(1)]_1 \times [SU(2) \times U(1)]_2 \subset SU(5)$:

$$Q_1^a = \frac{1}{2}\begin{pmatrix} \sigma^a & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \mathbf{0}_{2\times 2} \end{pmatrix} \qquad Q_2^a = \frac{1}{2}\begin{pmatrix} \mathbf{0}_{2\times 2} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\sigma^{a*} \end{pmatrix}$$

$$Y_1 = \frac{1}{10}\mathrm{diag}(3,3,-2,-2,-2) \qquad Y_2 = \frac{1}{10}\mathrm{diag}(2,2,2,-3,-3)$$

Littlest Higgs with T-parity

[Cheng, Low '03]

New particles at TeV scale coupling to SM particles ⇒ tension with EW precision tests

⇝ T-parity discrete symmetry under which SM (most of new) particles are even (odd)

Gauge sector

$G_1 \overset{\mathrm{T}}{\longleftrightarrow} G_2$ with $G_j = (W_j^a, B_j)$ gauge bosons of $[SU(2)\times U(1)]_{j=1,2}$

and $g \equiv g_1 = g_2$, $g' \equiv g'_1 = g'_2$

T-even: $B, W^3(\gamma, Z), W^+, W^- \leftarrow \frac{1}{\sqrt{2}}(G_1 + G_2)$

T-odd: $A_H, Z_H, W_H^+, W_H^- \leftarrow \frac{1}{\sqrt{2}}(G_1 - G_2)$

$$\mathcal{L}_G = \sum_{j=1}^{2}\left[-\frac{1}{2}\mathrm{Tr}\left(\widetilde{W}_{j\mu\nu}\widetilde{W}_j^{\mu\nu}\right) - \frac{1}{4}B_{j\mu\nu}B_j^{\mu\nu}\right]$$

Scalar sector

$\Pi \overset{\mathrm{T}}{\longrightarrow} -\Omega\Pi\Omega$, where $\Omega = \mathrm{diag}(-1,-1,1,-1,-1)$

T-even: SM H doublet (h, π^0, π^+, π^-)

T-odd: the others $(\eta, \omega^0, \omega^+, \omega^-, \Phi)$

$$\mathcal{L}_S = \frac{f^2}{8}\mathrm{Tr}\left[(D_\mu\Sigma)^\dagger(D^\mu\Sigma)\right] \supset \text{gauge boson masses}$$

with $D_\mu\Sigma = \partial_\mu\Sigma - \sqrt{2}\mathrm{i}\sum_{j=1}^{2}\left[gW_{j\mu}^a(Q_j^a\Sigma + \Sigma Q_j^{aT}) - g'B_{j\mu}(Y_j\Sigma + \Sigma Y_j^T)\right]$

(a) **Introduce** $SU(2)_L$ *doublets*: l_{1L}, l_{2L}, l_{HR} and $\tilde{l}^c_L$ in

$$\Psi_1[\bar{\mathbf{5}}] = \begin{pmatrix} -i\sigma^2 l_{1L} \\ 0 \\ 0 \end{pmatrix} \quad \Psi_2[\mathbf{5}] = \begin{pmatrix} 0 \\ 0 \\ -i\sigma^2 l_{2L} \end{pmatrix} \quad \Psi_R = \begin{pmatrix} -i\sigma^2 \tilde{l}^c_L \\ \chi_R \\ -i\sigma^2 l_{HR} \end{pmatrix}$$

T-even	**Standard** $l_L = \frac{1}{\sqrt{2}}(l_{1L} - l_{2L}) = (\nu_L \quad \ell_L)^T$	
T-odd	**Mirror** $l_{HL} = \frac{1}{\sqrt{2}}(l_{1L} + l_{2L}) = (\nu_{HL} \quad \ell_{HL})^T$ $l_{HR} = (\nu_{HR} \quad \ell_{HR})^T$	**Mirror partners** (decouple?) $\tilde{l}^c_L = (\tilde{\nu}^c_L \quad \tilde{\ell}^c_L)^T$

▷ To obtain (heavy) mirror fermion masses *preserving* gauge and T

$$\mathcal{L}_{Y_H} = -\kappa f \left(\overline{\Psi}_2 \xi + \overline{\Psi}_1 \Sigma_0 \xi^\dagger\right) \Psi_R + \text{h.c.} \qquad \xi = e^{i\Pi/f}$$

(b) **Introduce** (light) *standard fermion singlets* ℓ_R with mass terms *preserving* gauge and T

[Chen, Tobe, Yuan '06]

$$\mathcal{L}_Y = \frac{i\lambda_\ell}{2\sqrt{2}} f \epsilon_{ij}\epsilon_{xyz}\left[(\overline{\Psi}'_2)_x \Sigma_{iy}\Sigma_{jz} X + \text{T-transformed}\right] \ell_R + \text{h.c.} \qquad \Psi'_2 = \begin{pmatrix} 0 \\ 0 \\ l_{2L} \end{pmatrix}, \; X = (\Sigma_{33})^{-\frac{1}{4}}$$

▷ So far, masses from Yukawa couplings:

$$\mathcal{L}_Y \supset -\underbrace{\frac{\lambda_\ell}{\sqrt{2}}v}_{\text{lepton masses}}\ \overline{\ell_L}\ell_R + \text{h.c.}$$

$$\mathcal{L}_{Y_H} \supset -\underbrace{\sqrt{2}\kappa f}_{\text{mirror lepton masses}}\ \overline{l_{HL}}l_{HR} + \text{h.c.}$$

(c) To provide mirror partners $\tilde{l}^c_L$ (RH) with (heavy) masses **introduce** $\tilde{l}^c_R = (\tilde{\nu}^c_R,\ \tilde{\ell}^c_R)^T$ (LH) in

$$\Psi_L = \begin{pmatrix} -\mathrm{i}\sigma^2\tilde{l}^c_R \\ 0 \\ 0 \end{pmatrix} \sim \Psi_R \qquad \boxed{\mathcal{L}_M = -M\,\overline{\tilde{l}_R}\,\tilde{l}_L + \text{h.c.}}$$

(d) The gauge interactions with fermions are fixed!

$$\mathcal{L}_F = \mathrm{i}\overline{\Psi}_1\gamma^\mu D^*_\mu\Psi_1 \ + \ \mathrm{i}\overline{\Psi}_2\gamma^\mu D_\mu\Psi_2 + \mathrm{i}\overline{\Psi}_R\gamma^\mu\left(\partial_\mu + \tfrac{1}{2}\xi^\dagger(D_\mu\xi) + \tfrac{1}{2}\xi(\Sigma_0 D^*_\mu\Sigma_0\xi^\dagger)\right)\Psi_R + (\Psi_R \to \Psi_L)$$

with $D_\mu = \partial_\mu - \sqrt{2}\mathrm{i}g(W^a_{1\mu}Q^a_1 + W^a_{2\mu}Q^a_2) + \sqrt{2}\mathrm{i}g'\left(Y_1 B_{1\mu} + Y_2 B_{2\mu}\right)$

include $\mathcal{O}(v^2/f^2)$ couplings to Goldstones that render one-loop amplitudes (vid. Z-penguins) UV finite

[del Águila, JI, Jenkins '09]

(e) The gauge interactions of standard right-handed *singlets* (ℓ_R)

$$\mathcal{L}'_F = \mathrm{i}\bar{\ell}_R\gamma^\mu(\partial_\mu + \mathrm{i}g' y_\ell B_\mu)\ell_R \qquad y_\ell = -1$$

enlarging $SU(5)$ with two extra $U(1)$'s to assign proper hypercharges to *all* fermions

[Goto, Okada, Yamamoto '09]

LHT | (Lepton) flavor mixing

- T parity ⇒ SM (T-even) fermions do not mix with T-odd fermions
- 3 generations: $\boldsymbol{\lambda_\ell}$, $\boldsymbol{\kappa}$ and $\boldsymbol{M}$ are matrices in flavor space

$$\frac{\boldsymbol{\lambda_\ell}}{\sqrt{2}} v = V_L^\ell \,\mathrm{diag}(m_{\ell i})\, V_R^{\ell\dagger}$$

$$\begin{pmatrix} \sqrt{2}\boldsymbol{\kappa} f & 0 \\ 0 & \boldsymbol{M} \end{pmatrix} = \begin{pmatrix} V_L^H & 0 \\ 0 & \widetilde{V}_L \end{pmatrix} \begin{pmatrix} \mathrm{diag}(m_{\ell_{Hi}}) & 0 \\ 0 & \mathrm{diag}(m_{\tilde{\ell}_i}) \end{pmatrix} \begin{pmatrix} V_R^{H\dagger} & 0 \\ 0 & \widetilde{V}_R^\dagger \end{pmatrix}$$

- This leads to flavor mixings in both charged and neutral currents encoded in

$$\boldsymbol{V} \equiv V_L^{H\dagger} V_L^\ell \qquad\qquad \boldsymbol{W} \equiv \widetilde{V}_L^T V_R^H$$

misalignment between l_H and l — *misalignment* between $\tilde{l}^c$ and l_H

$$\overline{l_{HL}} \not{G}_H l_L \to \overline{l_{HL}} \boldsymbol{V} \not{G}_H l_L \qquad \overline{\tilde{l}_L^c} \not{G} l_{HR} \to \overline{\tilde{l}_L^c} \boldsymbol{W} \not{G} l_{HR} \quad (W^\pm \text{ only})$$

$$\overline{l_{HL}} \boldsymbol{\lambda_\ell} \ell_R \to \overline{l_{HL}} (\boldsymbol{V} \lambda_\ell) \ell_R \qquad \overline{\tilde{l}_L^c} \boldsymbol{\kappa}^\dagger l_{HL} \to \overline{\tilde{l}_L^c} (\boldsymbol{W} \kappa) l_{HL}$$

$$\overline{l_{HR}} \boldsymbol{\kappa}^\dagger l_L \to \overline{l_{HR}} (\kappa \boldsymbol{V}) l_L \qquad \overline{\tilde{l}_L^c} \boldsymbol{\kappa}^\dagger l_L \to \overline{\tilde{l}_L^c} (\boldsymbol{W} \kappa \boldsymbol{V}) l_L$$

LHT | (Lepton) flavor mixing

- ▷ W is a source of LFV, so far ignored assuming that mirror partners decouple.

- ▷ In $h \to \bar{\ell}_1 \ell_2$ [1705.08827]
  - The mirror partners $\tilde{l}^c$ do NOT decouple!
  - They are needed to make the amplitude UV finite!
  - The complex $SU(2)$ triplet of physical Goldstones Φ $(\Phi^0, \Phi^P, \Phi^\pm, \Phi^{\pm\pm})$ plays a role

- ▷ In other LF changing processes ($Z \to \bar{\ell}_1 \ell_2$, $\ell_1 \to \ell_2 \gamma$, $\ell \to \ell_1 \ell_2 \bar{\ell}_3$, μN $\to$ eN) [1901.07058]
  - The mirror partners $\tilde{l}^c$ decouple!
  - Φ contributions are subleading in v^2/f^2 when $\tilde{l}^c$ decouple

- ▷ Study the phenomenology of the **full lepton flavor of LHT**

▷ To simplify the discussion consider just **2-family mixing**:

$$V = \begin{pmatrix} \cos\theta_V & \sin\theta_V \\ -\sin\theta_V & \cos\theta_V \end{pmatrix} \qquad W = \begin{pmatrix} \cos\theta_W & \sin\theta_W \\ -\sin\theta_W & \cos\theta_W \end{pmatrix}$$

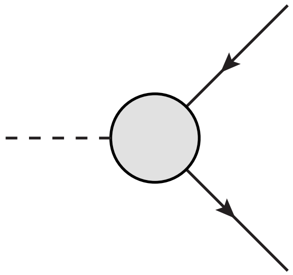
$$m_{\ell_{H1}}\,, \quad m_{\ell_{H2}} \qquad \Longleftrightarrow \qquad \overline{m}_{\ell_H} \equiv \sqrt{m_{\ell_{H1}} m_{\ell_{H2}}}\,, \qquad \delta_{\ell_H} \equiv \frac{m^2_{\ell_{H2}} - m^2_{\ell_{H1}}}{\overline{m}^2_{\ell_H}}$$

$$m_{\tilde{\nu}^c_1}\,, \quad m_{\tilde{\nu}^c_2} \qquad \Longleftrightarrow \qquad \overline{m}_{\tilde{\nu}^c} \equiv \sqrt{m_{\tilde{\nu}^c_1} m_{\tilde{\nu}^c_2}}\,, \qquad \delta_{\tilde{\nu}^c} \equiv \frac{m^2_{\tilde{\nu}^c_2} - m^2_{\tilde{\nu}^c_1}}{\overline{m}^2_{\tilde{\nu}^c}}$$

▷ LFV effects *vanish* for zero mixings or degenerate fermions

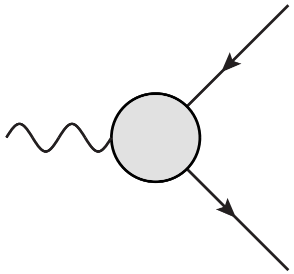
LHT **Lepton flavor changing processes** (T–odd particles running in the loop)

$h \to \bar{\ell}_1 \ell_2$

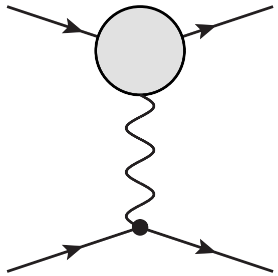


$Z \to \bar{\ell}_1 \ell_2$

$\ell_1 \to \ell_2 \gamma$



$\ell \to \ell_1 \ell_2 \bar{\ell}_3$



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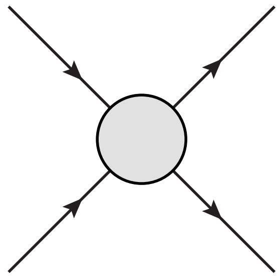
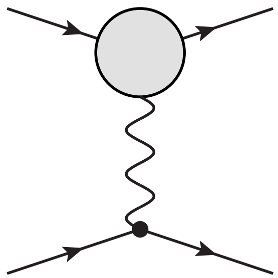


+ crossed ($\ell_1 \leftrightarrow \ell_2$)

$\mu N \to eN$



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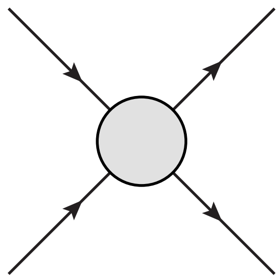
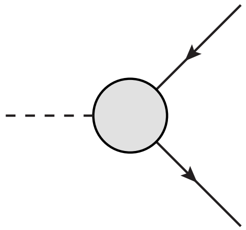


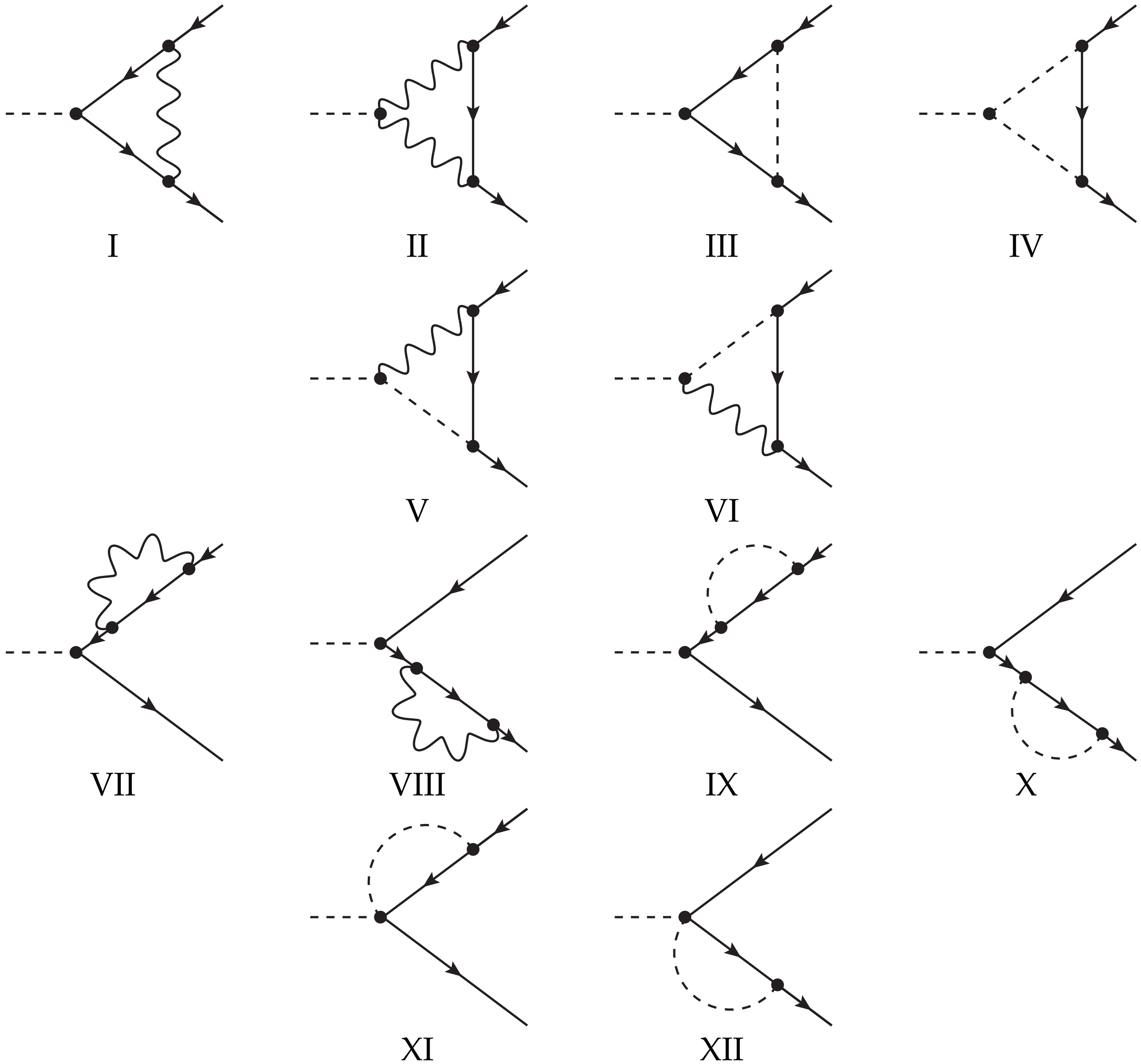
($q = u, d$)

LHT | One-loop contributions | $h \to \bar{\ell}_1 \ell_2$

('t Hooft-Feynman gauge)



V	S	F
	h	ℓ
$W_H^\pm$	$\omega^\pm$	ν_H
Z_H	ω^0	ℓ_H
A_H	η	$\tilde{\nu}^c$
	Φ^0	$\tilde{\ell}^c$
	Φ^P	
	$\Phi^\pm$	
	$\Phi^{\pm\pm}$	

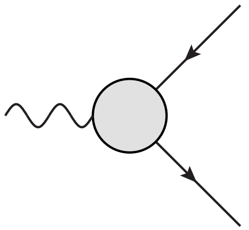


$\Leftarrow$ included

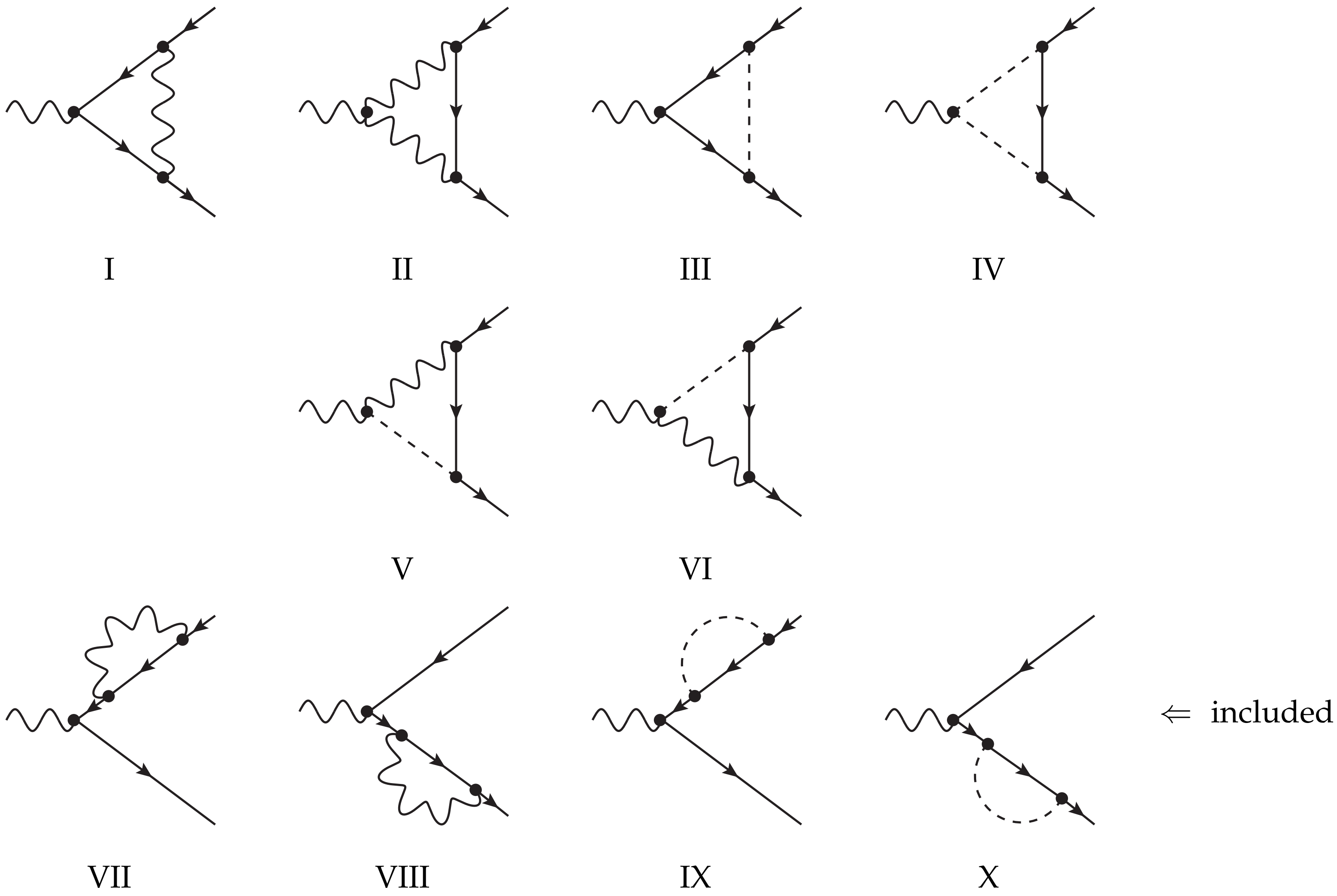
$\Leftarrow$ not in $V\bar{f}f$

LHT | **One-loop contributions** | $Z \to \bar{\ell}_1 \ell_2$, $\ell_1 \to \ell_2 \gamma$ (& penguins)

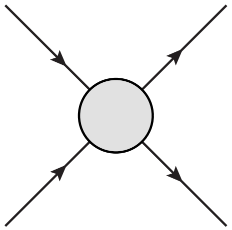
('t Hooft-Feynman gauge)



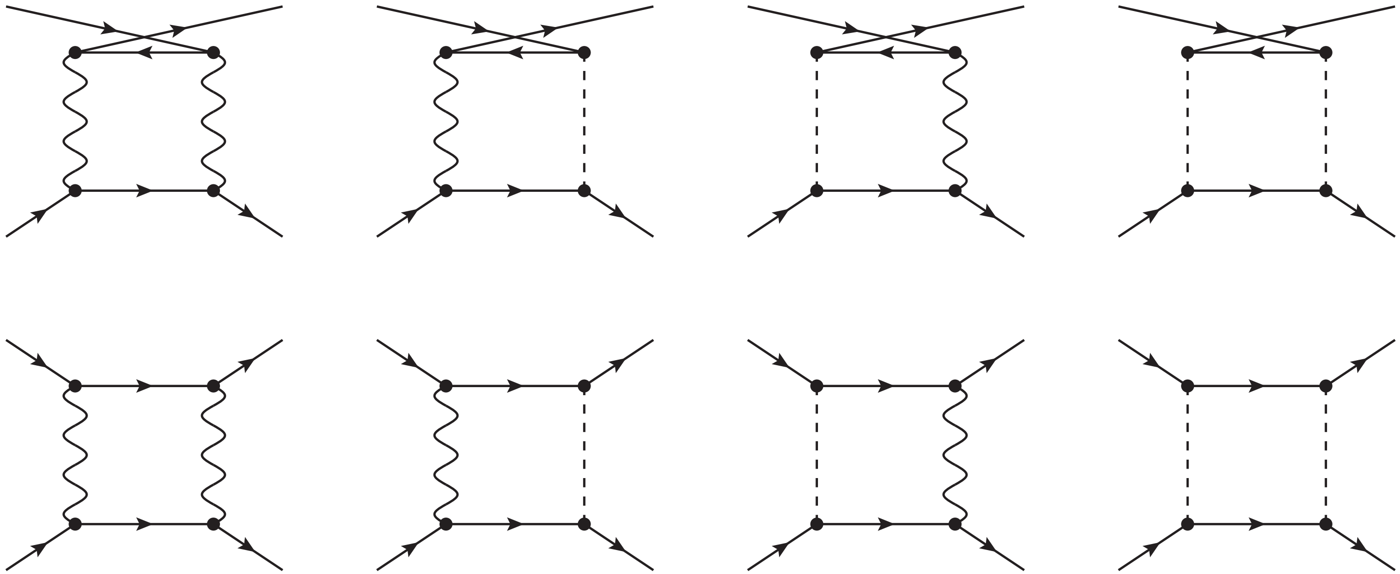
V	S	F
γ, Z, $W^{\pm}$		ℓ
$W_H^{\pm}$	$\omega^{\pm}$	ν_H
Z_H	ω^0	ℓ_H
A_H	η	$\tilde{\nu}^c$
	Φ^0	$\tilde{\ell}^c$
	Φ^P	
	$\Phi^{\pm}$	
	$\Phi^{\pm\pm}$	



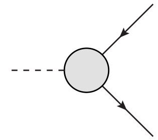
('t Hooft-Feynman gauge)



V	S	F
γ, Z, $W^\pm$		ℓ, u, d
$W_H^\pm$	$\omega^\pm$	ν_H
Z_H	ω^0	ℓ_H
A_H	η	$\tilde{\nu}^c$
	Φ^0	$\tilde{\ell}^c$
	Φ^P	u_H
	$\Phi^\pm$	d_H
	$\Phi^{\pm\pm}$	$\tilde{u}$ $(Q = 2/3)$
		$\tilde{x}$ $(Q = 5/3)$



LHT **Amplitude** $h \to \bar\tau\mu$



$$\mathcal{M}(h\to\mu\bar\tau) = \bar u(p_2)\left(\boxed{\frac{m_\mu}{v}}\,\boxed{c_L}\,P_L + \boxed{\frac{m_\tau}{v}}\,\boxed{c_R}\,P_R\right)v(p_1)$$

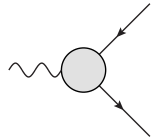
$$c_{L(R)} = \boxed{\frac{g^2}{16\pi^2}}\boxed{\frac{v^2}{f^2}}\left\{\sum_{i=1}^{3} V^\dagger_{\mu i}V_{i\tau}\,\mathcal{F}_h(m_{\ell_{Hi}}, m_{W_H}, m_{A_H}, m_\Phi)\right.$$
$$\left. + \sum_{i,j,k=1}^{3} V^\dagger_{\mu i}\frac{m_{\ell_{Hi}}}{m_{W_H}}W^\dagger_{ij}W_{jk}\frac{m_{\ell_{Hk}}}{m_{W_H}}V_{k\tau}\,\mathcal{G}_h(m_{\tilde\nu^c_j}, m_{\ell_{Hk(i)}}, m_\Phi)\right\}$$

(terms of $\mathcal{O}(1)$ cancel!) $\quad \mathcal{G}_h \xrightarrow[m_{\nu^c_j}\to\infty]{} \ln m^2_{\nu^c_i} \quad [h\bar\nu_H\tilde\nu^c]$

$$c_{L(R)} \sim \frac{g^2}{16\pi^2}\frac{v^2}{f^2}\left\{\boxed{\sin 2\theta_V}\,[\mathcal{F}_h(m_{\ell_{H1}}) - \mathcal{F}_h(m_{\ell_{H2}})]\,,\ \underbrace{\frac{\overline{m}^2_{\ell_H}}{M^2_{W_H}}\boxed{\sin 2\theta_W}\ln\frac{m^2_{\tilde\nu^c_1}}{m^2_{\tilde\nu^c_2}}}_{m_{\tilde\nu^c_i}\to\infty\ (\theta_V=\delta_{\ell_H}=0)}\right\}$$

$\Rightarrow$ non-decoupling of $\tilde l^c_i$!!

LHT | **Amplitudes** | $Z \to \bar{\ell}_1 \ell_2$, $\ell_1 \to \ell_2 \gamma$



$$i\Gamma^\mu_\gamma = ie\left[F_L^\gamma(Q^2)\gamma^\mu P_L + iF_M^\gamma(Q^2)(1+\gamma_5)\sigma^{\mu\nu}q_\nu\right] \qquad F_L^\gamma \propto \frac{1}{16\pi^2}\frac{Q^2}{f^2} \qquad \overbrace{F_M^\gamma \propto \frac{1}{16\pi^2}\frac{m_\ell}{f^2}}^{\text{UV finite}}$$

$$i\Gamma^\mu_Z = ie\left[F_L^Z(Q^2)\gamma^\mu P_L + iF_M^Z(Q^2)(1+\gamma_5)\sigma^{\mu\nu}q_\nu\right] \qquad F_L^Z \propto \frac{g^2}{16\pi^2}\frac{v^2}{f^2} \qquad F_M^Z \propto \frac{g^2}{16\pi^2}\frac{m_\ell}{f^2}$$

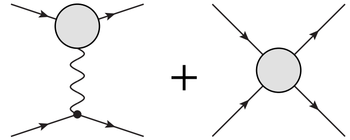
$F_L^\gamma(0) = 0$: $\ell_1 \to \ell_2\gamma$ is a dipole transition

$$F_{L,M}^{\gamma,Z} \sim \left\{ \sum_{i=1}^{3} V^\dagger_{\ell_2 i} V_{i\ell_1}\, \mathcal{F}_{L,M}^{\gamma,Z}(m_{\ell_{Hi}}, m_{W_H}, m_{A_H}, m_\Phi) + \sum_{i,j,k=1}^{3} V^\dagger_{\ell_2 i}\frac{m_{\ell_{Hi}}}{m_{W_H}} W^\dagger_{ij} W_{jk}\frac{m_{\ell_{Hk}}}{m_{W_H}} V_{k\ell_1}\, \mathcal{G}_{L,M}^{\gamma,Z}(m_{\tilde{\nu}_j^c}, m_\Phi)\right\}$$

(terms of $\mathcal{O}(1)$ cancel!) $\qquad \mathcal{G}_{L,M}^{\gamma,Z} \xrightarrow[m_{\nu_j^c}\to\infty]{} 0 \qquad [\cancel{V\bar{\nu}_H\tilde{\nu}^c}]$

$\Rightarrow$ there is decoupling of $\tilde{l}_i^c$!!

LHT | Amplitudes | $\ell \to \ell_1 \ell_2 \bar{\ell}_3$, μN $\to$ eN



$$\mathrm{i}\mathcal{M}_\gamma = \left(\frac{\mathrm{i}}{Q^2}\right) e \left[\overline{u}(p_1)\Gamma^\mu_\gamma u(p)\right] \left[\overline{u}(p_2)\gamma_\mu v(p_3)\right] - (\ell_1 \leftrightarrow \ell_2)$$

$$\mathrm{i}\mathcal{M}_Z = \left(-\frac{\mathrm{i}}{M_Z^2}\right) e \left[\overline{u}(p_1)\Gamma^\mu_Z u(p)\right] \left[\overline{u}(p_2)\gamma_\mu \left(Z_L P_L + Z_R P_R\right) v(p_3)\right] - (\ell_1 \leftrightarrow \ell_2)$$

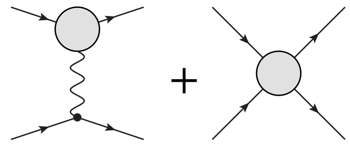
$$\mathrm{i}\mathcal{M}_{\text{box}} = \mathrm{i}e^2 B_L \left[\overline{u}(p_1)\gamma^\mu P_L u(p)\right] \left[\overline{u}(p_2)\gamma_\mu P_L v(p_3)\right] - (\ell_1 \leftrightarrow \ell_2)$$

- γ–penguins ($Q^2 \to 0$): $\quad A_L = \dfrac{F_L^\gamma}{Q^2} \quad A_R = \dfrac{2F_M^\gamma}{m_\ell} \quad \propto \dfrac{1}{16\pi^2}\dfrac{1}{f^2}$

- Z–penguins ($Q^2 \to 0$): $\quad F_{LL} = -\dfrac{F_L^Z}{M_Z^2} Z_L \quad F_{LR} = -\dfrac{F_L^Z}{M_Z^2} Z_R \quad \propto \dfrac{1}{16\pi^2}\dfrac{1}{f^2}$

- Boxes: $\quad B_L \quad \propto \dfrac{1}{16\pi^2}\dfrac{1}{f^2}$

LHT | **Amplitudes** | $\ell \to \ell_1 \ell_2 \bar{\ell}_3$, μN $\to$ eN



$$\mathrm{i}\mathcal{M}_\gamma = \left(\frac{\mathrm{i}}{Q^2}\right) e \left[\overline{u}(p_1)\Gamma^\mu_\gamma P_L u(p)\right] \left[\overline{u}(p_2)\gamma_\mu v(p_3)\right] - (\ell_1 \leftrightarrow \ell_2)$$

$$\mathrm{i}\mathcal{M}_Z = \left(-\frac{\mathrm{i}}{M_Z^2}\right) e \left[\overline{u}(p_1)\Gamma^\mu_Z P_L u(p)\right] \left[\overline{u}(p_2)\gamma_\mu \left(Z_L P_L + Z_R P_R\right) v(p_3)\right] - (\ell_1 \leftrightarrow \ell_2)$$

$$\mathrm{i}\mathcal{M}_{\mathrm{box}} = \mathrm{i}e^2 B_L \left[\overline{u}(p_1)\gamma^\mu P_L u(p)\right] \left[\overline{u}(p_2)\gamma_\mu P_L v(p_3)\right] - (\ell_1 \leftrightarrow \ell_2)$$

$$B_L \sim \left\{ \sum_{ij=1}^{3} V^\dagger_{\ell_1 i} V_{i\ell} V^\dagger_{\ell_2 j} V_{j\ell_3}\, \mathcal{F}_B(m_{\ell_{Hi}}, m_{\ell_{Hi}}, \ldots) + \sum_{ijpqrs=1}^{3} V^\dagger_{\ell_1 p} \frac{m_{\ell_{Hp}}}{m_{W_H}} W^\dagger_{pi} W_{iq} \frac{m_{\ell_{Hq}}}{m_{W_H}} V_{q\ell} V^\dagger_{\ell_2 r} \frac{m_{\ell_{Hr}}}{m_{W_H}} W^\dagger_{rj} W_{js} \frac{m_{\ell_{Hs}}}{m_{W_H}} V_{s\ell_3}\, \mathcal{G}_B(m_{\tilde{\nu}^c_i}, m_{\tilde{\nu}^c_j}, \ldots) \right\}$$

(UV finite)

LHT **UV divergences**

Highly non-trivial cancellations!

$$\left\{c_{L(R)}, \quad 2s_W c_W F_L^Z, \quad F_L^\gamma\right\} \quad \sim \quad \frac{1}{16\pi^2}\sum_i V_{\mu i}^\dagger V_{i\tau}\frac{m_{\ell_{Hi}}^2}{f^2}\left(\boxed{\frac{C_{\rm UV}^{(0)}}{\epsilon}}+\boxed{\frac{C_{\rm UV}^{(1)}}{\epsilon}}\frac{v^2}{f^2}\right)$$

$$\epsilon = 4 - D$$

LHT **UV divergences** $C_{\rm UV}^{(0)}$

$c_{L(R)}$	I	II	III	IV	V+VI	VII+VIII	IX+X	XI+XII	sum
ω, ν_H	–	–	•	•	–	–	1	-1	•
ω^0, ℓ_H	–	–	•	•	–	–	$\frac{1}{2}$	$-\frac{1}{2}$	•
η, ℓ_H	–	–	•	•	–	–	$\frac{1}{10}$	$-\frac{1}{10}$	•
all	–	–	•	•	–	–	$\frac{8}{5}$	$-\frac{8}{5}$	•

$2s_W c_W F_L^Z$	I	II	III	IV	V+VI	VII+VIII	IX+X	sum
W_H, ν_H	0	0	–	–	–	–	0	0
W_H, ω, ν_H	–	–	–	–	0	–	–	0
ω, ν_H	–	–	$\frac{1}{2}$	$-1+s_W^2$	–	–	$\frac{1}{2}-s_W^2$	•
Z_H, ℓ_H	0	•	–	–	–	–	0	•
ω^0, ℓ_H	–	–	$-\frac{1}{4}+\frac{1}{2}s_W^2$	•	–	–	$\frac{1}{4}-\frac{1}{2}s_W^2$	•
A_H, ℓ_H	0	•	–	–	–	0	–	•
η, ℓ_H	–	–	$-\frac{1}{20}+\frac{1}{10}s_W^2$	0	–	–	$\frac{1}{20}-\frac{1}{10}s_W^2$	•
$\Phi, \tilde{\nu}^c$	–	–	$-\frac{1}{2}$	s_W^2	–	–	$\frac{1}{2}-s_W^2$	•
$\Phi^{++}, \tilde{\ell}^c$	–	–	$1-2s_W^2$	$-2+4s_W^2$	–	–	$1-2s_W^2$	•
all	0	0	$\frac{7}{10}-\frac{7}{5}s_W^2$	$-3+6s_W^2$	0	0	$\frac{23}{10}-\frac{23}{5}s_W^2$	•

F_L^γ	all	sum
all	•	•

• also finite part is zero

LHT | **UV divergences** | $C_{\rm UV}^{(1)}$ | $c_{L(R)}$

$c_{L(R)}$	I	II	III	IV	V+VI	VII+VIII	IX+X	XI+XII	sum	
W_H, ν_H	0	0	–	–	–	•	–	–	0	
W_H, ω, ν_H	–	–	–	–	0	–	–	–	0	$F\vert_{W_H}$
ω, ν_H	–	–	$\frac{1}{4}$	$-\frac{1}{8}$	–	–	$-\frac{1}{6}$	$\frac{5}{24}$	$\frac{1}{6}$	
Z_H, ℓ_H	•	0	–	–	–	•	–	–	0	
Z_H, ω^0, ℓ_H	–	–	–	–	0	–	–	–	0	$F\vert_{Z_H}$ ✓
ω^0, ℓ_H	–	–	•	$-\frac{1}{16}$	–	–	$-\frac{13}{48} + x_H \frac{c_W}{s_W}$	$\frac{7}{16} - x_H \frac{c_W}{s_W}$	$\frac{5}{48}$	
A_H, ℓ_H	•	0	–	–	–	•	–	–	0	
A_H, η, ℓ_H	–	–	–	–	0	–	–	–	0	$F\vert_{A_H}$ ✓
η, ℓ_H	–	–	•	$-\frac{1}{16}$	–	–	$-\frac{23}{240} - x_H \frac{s_W}{5c_W}$	$-\frac{17}{240} + x_H \frac{s_W}{5c_W}$	$-\frac{11}{48}$	
Z_H, A_H, ℓ_H	–	0	–	–	–	–	–	–	0	$F\vert_{Z_H A_H}$ ✓
ω^0, η, ℓ_H	–	–	–	$\frac{1}{8}$	–	–	–	–	$\frac{1}{8}$	
W_H, Φ, ν_H	–	–	–	–	0	–	–	–	0	
Φ, ν_H	–	–	•	•	–	–	$-\frac{1}{8}$	$\frac{1}{24}$	$-\frac{1}{12}$	
ω, Φ, ν_H	–	–	–	$\frac{1}{6}$	–	–	–	–	$\frac{1}{6}$	$F\vert_{\Phi\, l_H}$
ω^0, Φ^P, ℓ_H	–	–	–	$\frac{1}{24}$	–	–	–	–	$\frac{1}{24}$	
η, Φ^P, ℓ_H	–	–	–	$-\frac{1}{24}$	–	–	–	–	$-\frac{1}{24}$	
⇒ $\Phi, \tilde{\nu}^c, \nu_H$	–	–	$-\frac{1}{4}$	$\frac{1}{24}$	–	–	•	$-\frac{1}{24}$	$-\frac{1}{4}$	G ⇐
all	0	0	0	$\frac{1}{12}$	0	•	$-\frac{49}{120}$	$\frac{39}{120}$	0	$\mathcal{M}$

LHT **UV divergences** $C_{\rm UV}^{(1)}$ $2s_W c_W F_L^Z$

$2s_W c_W F_L^Z$	I	II	III	IV	V+VI	VII+VIII	IX+X	sum
W_H, ν_H	0	0	–	–	–	0	–	0
W_H, ω, ν_H	–	–	–	–	0	–	–	0
ω, ν_H	–	–	$-\frac{1}{8}$	$\frac{1}{8}$	–	–	0	0
Z_H, ℓ_H	0	•	–	–	–	0	–	0
ω^0, ℓ_H	–	–	$\frac{1}{16} - \frac{1}{8}s_W^2 - y_H \frac{c_W}{s_W}$	•	–	–	$-\frac{1}{16} + \frac{1}{8}s_W^2 + y_H \frac{c_W}{s_W}$	0
A_H, ℓ_H	0	•	–	–	–	0	–	0
η, ℓ_H	–	–	$\frac{1}{16} - \frac{1}{8}s_W^2 + y_H \frac{s_W}{5c_W}$	•	–	–	$-\frac{1}{16} + \frac{1}{8}s_W^2 - y_H \frac{s_W}{5c_W}$	0
$\Phi, \tilde{\nu}^c$	–	–	$\frac{1}{8}$	$-\frac{1}{8}$	–	–	•	0
$\Phi^{++}, \tilde{\ell}^c$	–	–	0	0	–	–	•	0
all	0	0	0	0	0	0	0	0

F_L^γ	I	II	III	IV	V+VI	VII+VIII	IX+X	sum
W_H, ν_H	•	0	–	–	–	0	–	0
W_H, ω, ν_H	–	–	–	–	0	–	–	0
ω, ν_H	–	–	•	$\frac{1}{2}$	–	–	$-\frac{1}{2}$	0
Z_H, ℓ_H	0	•	–	–	–	0	–	0
ω^0, ℓ_H	–	–	$\frac{1}{4}$	•	–	–	$-\frac{1}{4}$	0
A_H, ℓ_H	0	•	–	–	–	0	–	0
η, ℓ_H	–	–	$\frac{1}{100}\frac{s_W^2}{c_W^2}$	•	–	–	$-\frac{1}{100}\frac{s_W^2}{c_W^2}$	0
$\Phi, \tilde{\nu}^c$	–	–	•	$\frac{1}{2}$	–	–	$-\frac{1}{2}$	0
$\Phi^{++}, \tilde{\ell}^c$	–	–	-1	2	–	–	-1	0
all	0	0	$-\frac{3}{4}+\frac{1}{100}\frac{s_W^2}{c_W^2}$	3	0	0	$-\frac{9}{4}-\frac{1}{100}\frac{s_W^2}{c_W^2}$	0

- Reference inputs (*natural* values)

$$f = 1.5 \text{ TeV}$$
$$\overline{m}_{\ell_H} = \overline{m}_{\tilde{\nu}^c} = 1.0 \text{ TeV}$$
$$(m_{d_{Hi}} = m_{\tilde{u}_i} = 2.0 \text{ TeV})$$
$$\delta_{\ell_H} = \delta_{\tilde{\nu}^c} = 1$$
$$\theta_V = \theta_W = \frac{\pi}{4}$$

$$\Rightarrow \quad m_{W_H} = m_{Z_H} \approx g f = 960 \text{ GeV}$$
$$m_{A_H} \approx \frac{g' f}{\sqrt{5}} = 230 \text{ GeV}$$
$$m_\Phi = \frac{\sqrt{2} f}{v} m_h = 1080 \text{ GeV}$$

- Current limits

Branching Ratio	90% C.L.	Branching Ratio	90% C.L.
$\mu \to e\,\gamma$	4.2×10^{-13}	$\mu \to e\,e\,\bar{e}$	1.0×10^{-12}
Conversion Rate			
$\mu \to e$ (Au)	7.0×10^{-13}		
$\mu \to e$ (Ti)	4.3×10^{-12}		

Branching Ratio		Branching Ratio	
$\tau \to e\,\gamma$	3.3×10^{-8}	$\tau \to \mu\,\mu\,\bar{e}$	1.7×10^{-8}
$\tau \to \mu\,\gamma$	4.4×10^{-8}	$\tau \to e\,e\,\bar{\mu}$	1.5×10^{-8}
		$\tau \to \mu\,e\,\bar{e}$	1.8×10^{-8}
		$\tau \to e\,\mu\,\bar{\mu}$	2.7×10^{-8}
		$\tau \to e\,e\,\bar{e}$	2.7×10^{-8}
		$\tau \to \mu\,\mu\,\bar{\mu}$	2.1×10^{-8}

Branching Ratio	95% C.L.	Branching Ratio	95% C.L.
$Z \to \mu\,e$	7.3×10^{-7}	$h \to \mu\,e$	3.5×10^{-4}
$Z \to \tau\,e$	9.8×10^{-6}	$h \to \tau\,e$	6.2×10^{-3}
$Z \to \tau\,\mu$	1.2×10^{-5}	$h \to \tau\,\mu$	2.5×10^{-3}

LHT flavor phenomenology | LFV decays

- LHT predictions for **reference inputs**

Branching Ratio		Branching Ratio	
$\mu \to e\,\gamma$	4.3×10^{-9}	$\mu \to e\,e\,\bar{e}$	2.5×10^{-11}
Conversion Rate			
$\mu \to e$ (Au)	3.8×10^{-9}		
$\mu \to e$ (Ti)	3.3×10^{-9}		

✓ constrained

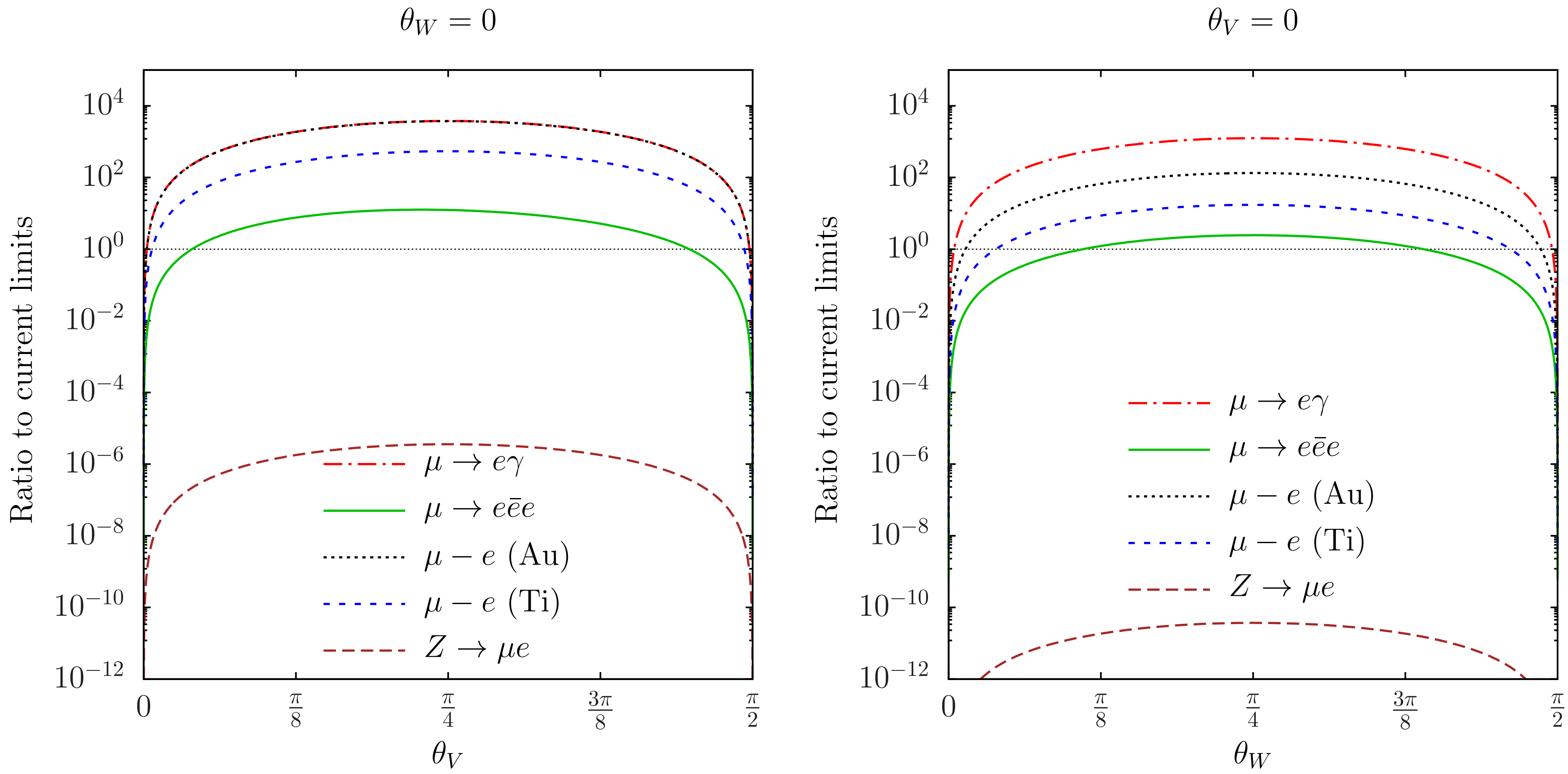
Branching Ratio		Branching Ratio	
$\tau \to e\,\gamma$	7.3×10^{-10}	$\tau \to \mu\,\mu\,\bar{e}$	0
$\tau \to \mu\,\gamma$	7.3×10^{-10}	$\tau \to e\,e\,\bar{\mu}$	0
		$\tau \to \mu\,e\,\bar{e}$	8.2×10^{-12}
		$\tau \to e\,\mu\,\bar{\mu}$	2.2×10^{-12}
		$\tau \to e\,e\,\bar{e}$	7.4×10^{-12}
		$\tau \to \mu\,\mu\,\bar{\mu}$	1.4×10^{-12}

– unconstrained by current expts

Branching Ratio		Branching Ratio	
$Z \to \mu\,e$	2.7×10^{-12}	$h \to \mu\,e$	1.2×10^{-15}
$Z \to \tau\,e$	2.7×10^{-12}	$h \to \tau\,e$	3.2×10^{-13}
$Z \to \tau\,\mu$	2.7×10^{-12}	$h \to \tau\,\mu$	3.2×10^{-13}

☹ hopeless

- **Parameter scans**

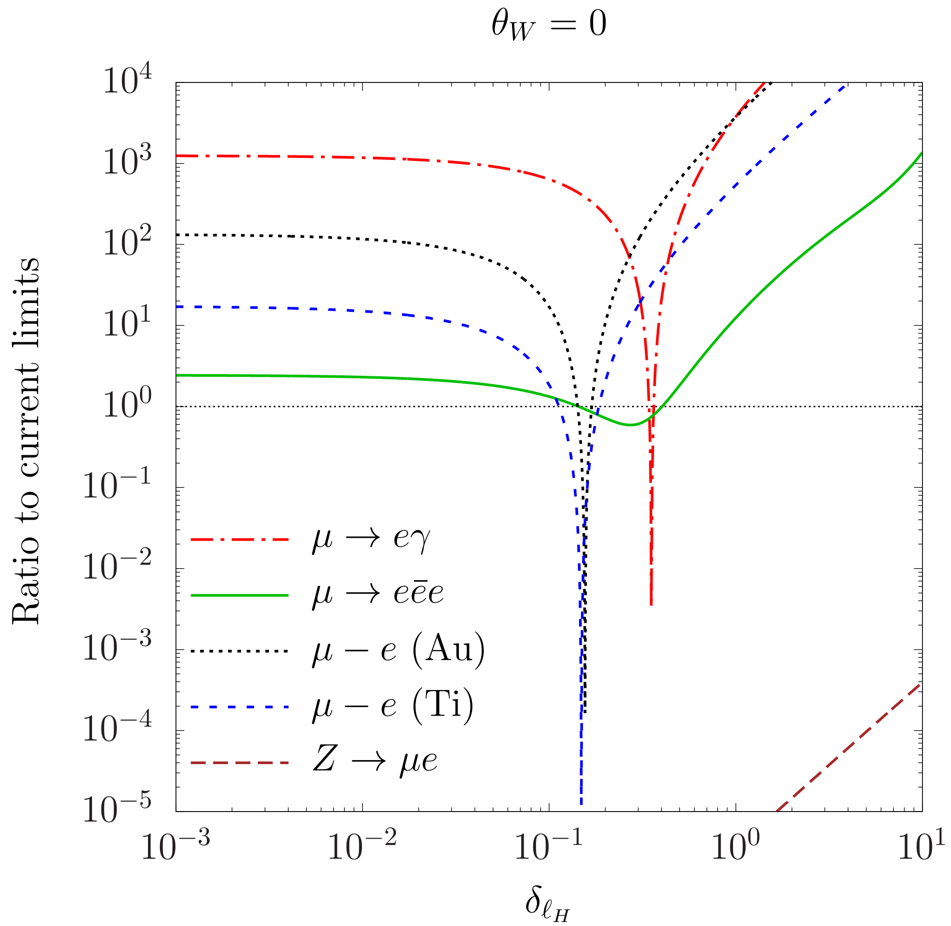


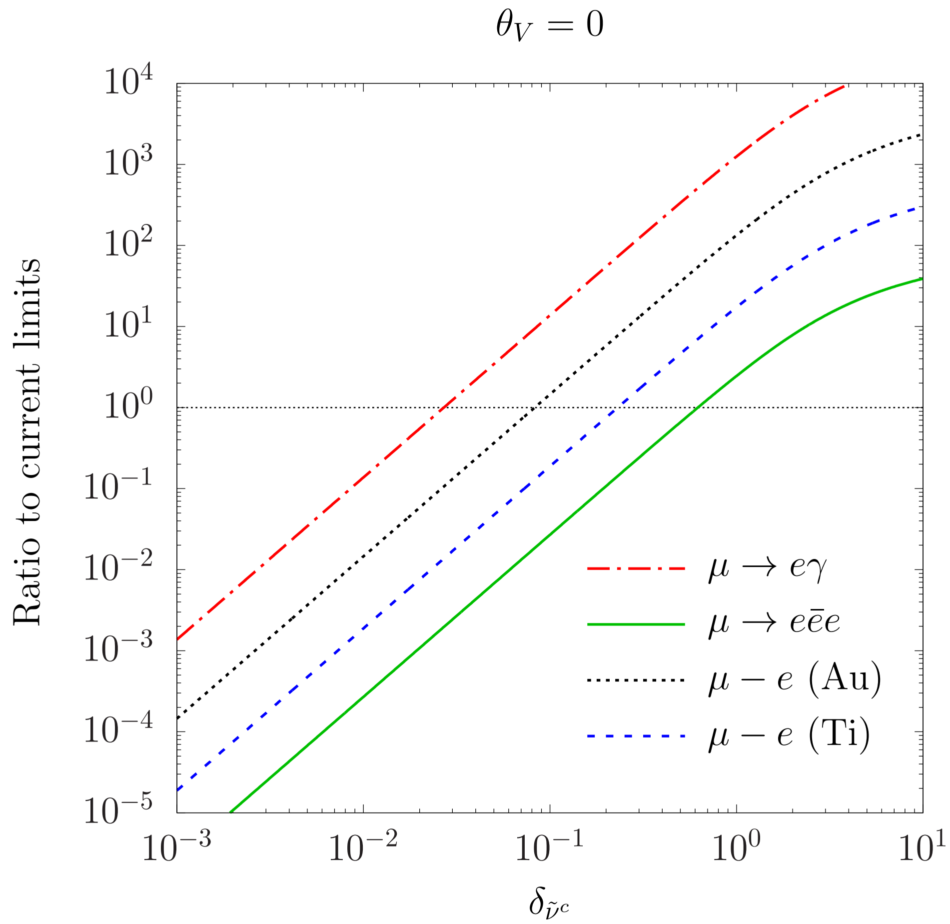
▷ No LFV for $\theta_V = \theta_W = 0$, maximal for $\theta_{V,W} = \frac{\pi}{4}$

▷ Z and h decay rates are very small

▷ $\mu \to e\gamma$ and $\mu - e$ (Au) are most sensitive

- **Parameter scans**

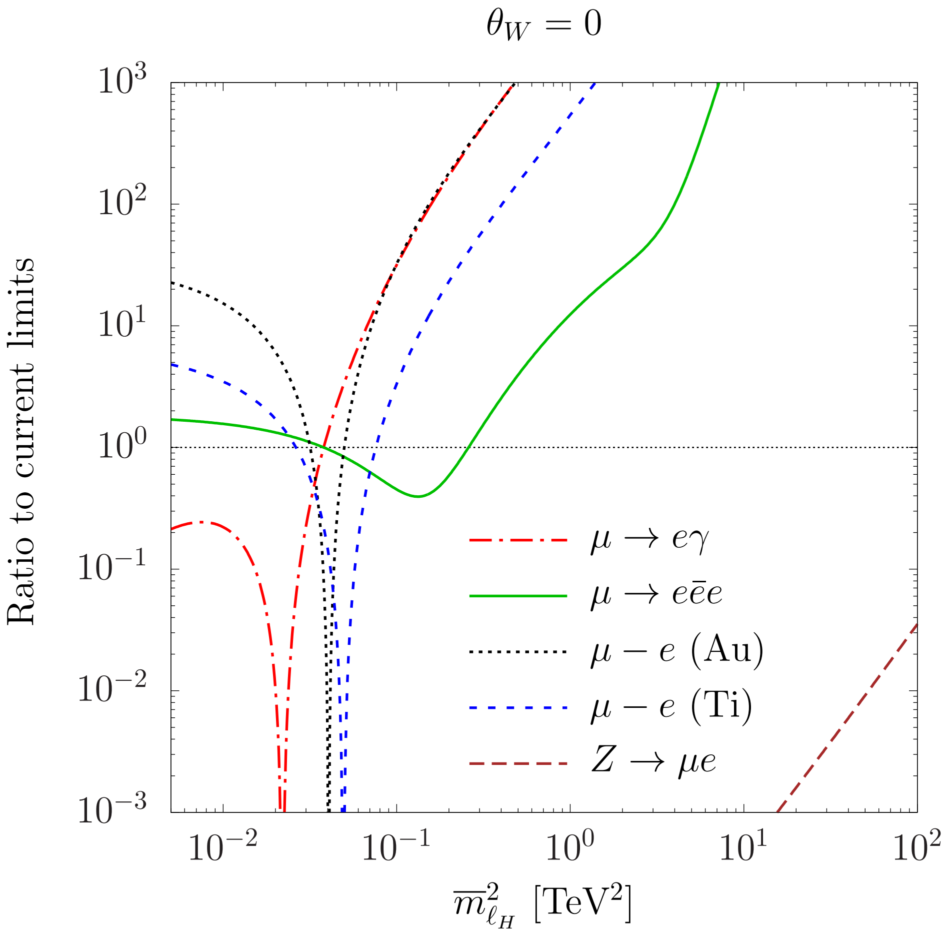


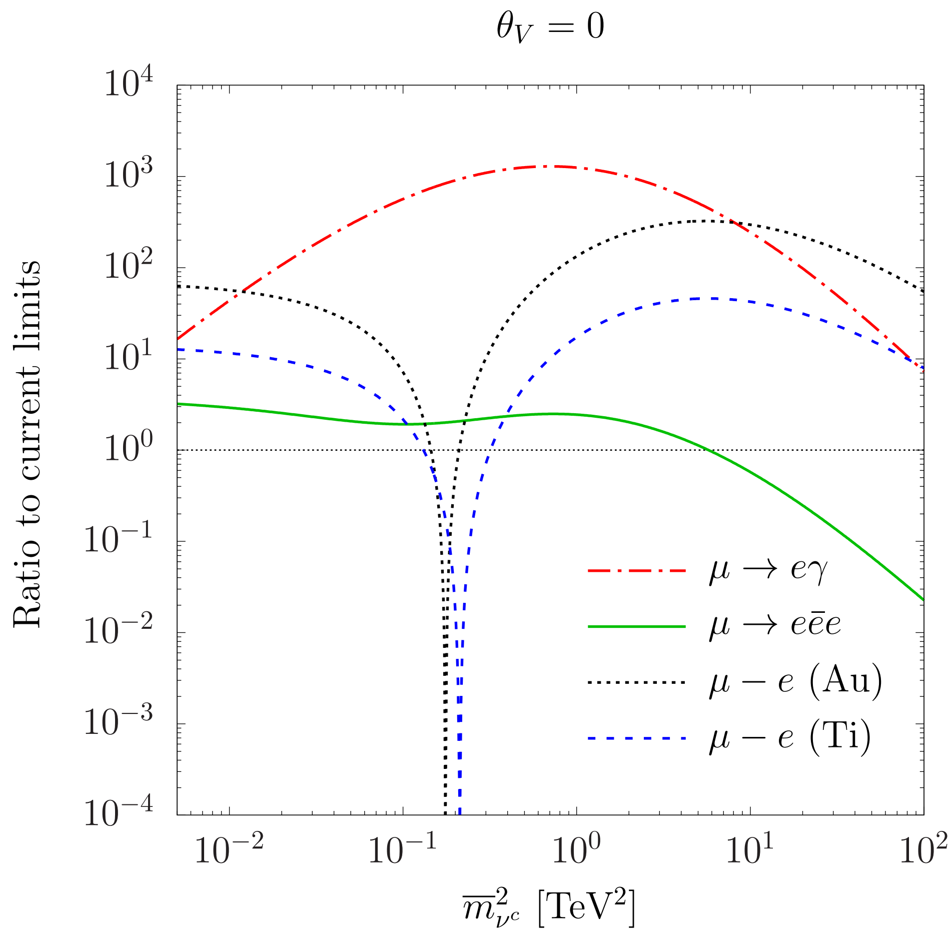


∇ ∇

$$\lim_{\delta_{\ell_H}\to 0} \mathcal{M}(\theta_W = 0) \sim \sin 2\theta_V [\mathcal{G}(m_{\nu_1^c}) - \mathcal{G}(m_{\nu_2^c})] \qquad \lim_{\delta_{\tilde{\nu}^c}\to 0} \mathcal{M}(\theta_V = 0) \sim \sin 2\theta_W [\mathcal{G}(m_{\nu_1^c}) - \mathcal{G}(m_{\nu_2^c})] \to 0$$

- **Parameter scans**

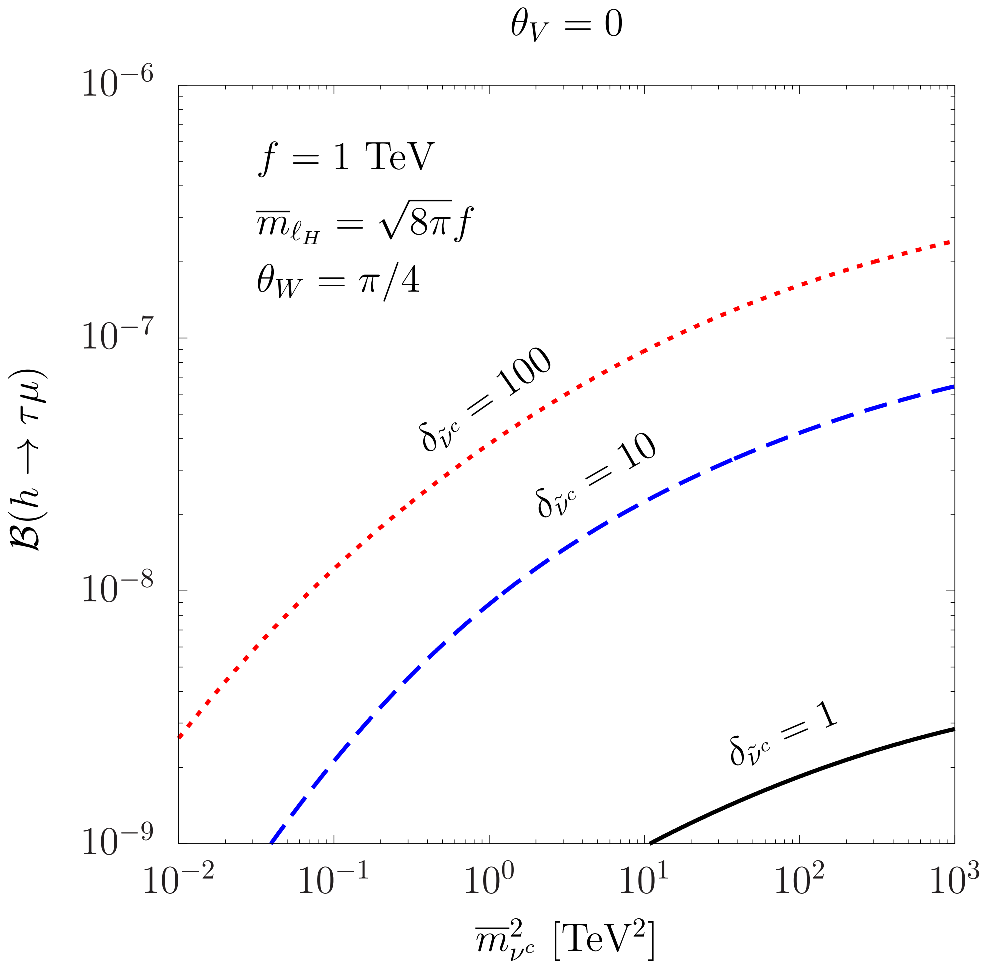




$\bigtriangledown$ $\bigtriangledown$

For large $\overline{m}_{\ell_H} = \sqrt{2}\overline{\kappa} f$: non perturbative coupling! For large $\overline{m}_{\tilde{\nu}^c}$: decoupling except h decays

(Current limit [LHC]: $\mathcal{B}(h \to \tau\mu) < 2.5 \times 10^{-3}$)



$$m^2_{\tilde{\nu}^c_1} = \overline{m}^2_{\tilde{\nu}^c} \frac{\sqrt{4 + \delta^2_{\tilde{\nu}^c}} - \delta_{\tilde{\nu}^c}}{2} \xrightarrow[\delta_{\tilde{\nu}^c} \gg 1]{} \overline{m}^2_{\tilde{\nu}^c} \times \frac{1}{\delta_{\tilde{\nu}^c}}$$

$$m^2_{\tilde{\nu}^c_2} = \overline{m}^2_{\tilde{\nu}^c} \frac{\sqrt{4 + \delta^2_{\tilde{\nu}^c}} + \delta_{\tilde{\nu}^c}}{2} \xrightarrow[\delta_{\tilde{\nu}^c} \gg 1]{} \overline{m}^2_{\tilde{\nu}^c} \times \delta_{\tilde{\nu}^c}$$

LHT flavor phenomenology | LFV decays | Generic constraints

- Current and Future sensitivities

$$\text{Observables} \propto \frac{\sin^2 2\theta}{f^4}$$

▷ $\mu - e$ transitions:

*precision = limit/prediction

Process	Experiment	Current precision*	Sensitivity improvement	[$\sin 2\theta = 1$] f [TeV] >		[$f = 1.5$ TeV] Mixing angle $< \times 10^{-2}$	
$\mu \to e\,\gamma$	[MEG]	$\mathbf{10^{-4}}$	$10 - 500$	**15**	$27 - 71$	**1**	$0.3 - 0.04$
$\mu \to e\,e\,\bar{e}$	[Mu3e]	0.04	$200 - 10^4$	3.4	$13 - 34$	20	$1 - 0.3$
$\mu \to e$ (Al)	[Mu2e]	10^{-3}	$\mathbf{10^4 - 10^5}$	8.4	**84 – 150**	3	**0.03 – 0.01**
$\mu \to e$ (Al)	[COMET]	10^{-3}	$10^2 - 10^4$	8.4	$27 - 84$	3	$0.3 - 0.03$

▷ $\tau - e$ and $\tau - \mu$ transitions:

Process	Experiment	Prediction	Current	Future	
$\mathcal{B}(\tau \to \ell\gamma)$	[BaBar]	7.3×10^{-10}	$\sim 10^{-8}$	$\mathbf{10^{-9} - 10^{-10}}$	[BelleII, LHCb]
$\mathcal{B}(\tau \to \ell\ell'\overline{\ell'})$	[Belle]	$\sim 10^{-12}$	$\sim 10^{-8}$	$10^{-9} - 10^{-10}$	[BelleII, LHCb]

$\Leftarrow$ ☺

- When γ **dipole** contribution ($|A_R|^2$) **dominates** then (kinematics):

$$\frac{\mathcal{B}(\tau\to\mu\mu\bar{\mu})}{\mathcal{B}(\tau\to\mu\gamma)}=\frac{\alpha}{3\pi}\left(2\ln\frac{m_\tau}{m_\mu}-\frac{13}{4}\right)\approx 2\times 10^{-3}$$

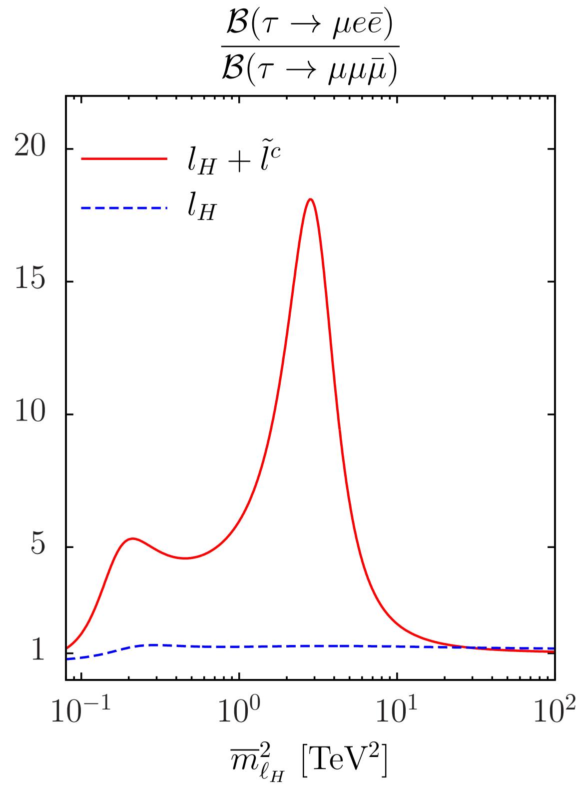
$$\frac{\mathcal{B}(\tau\to\mu e\bar{e})}{\mathcal{B}(\tau\to\mu\gamma)}=\frac{\alpha}{3\pi}\left(2\ln\frac{m_\tau}{m_e}-\frac{7}{2}\right)\approx 10^{-2}$$

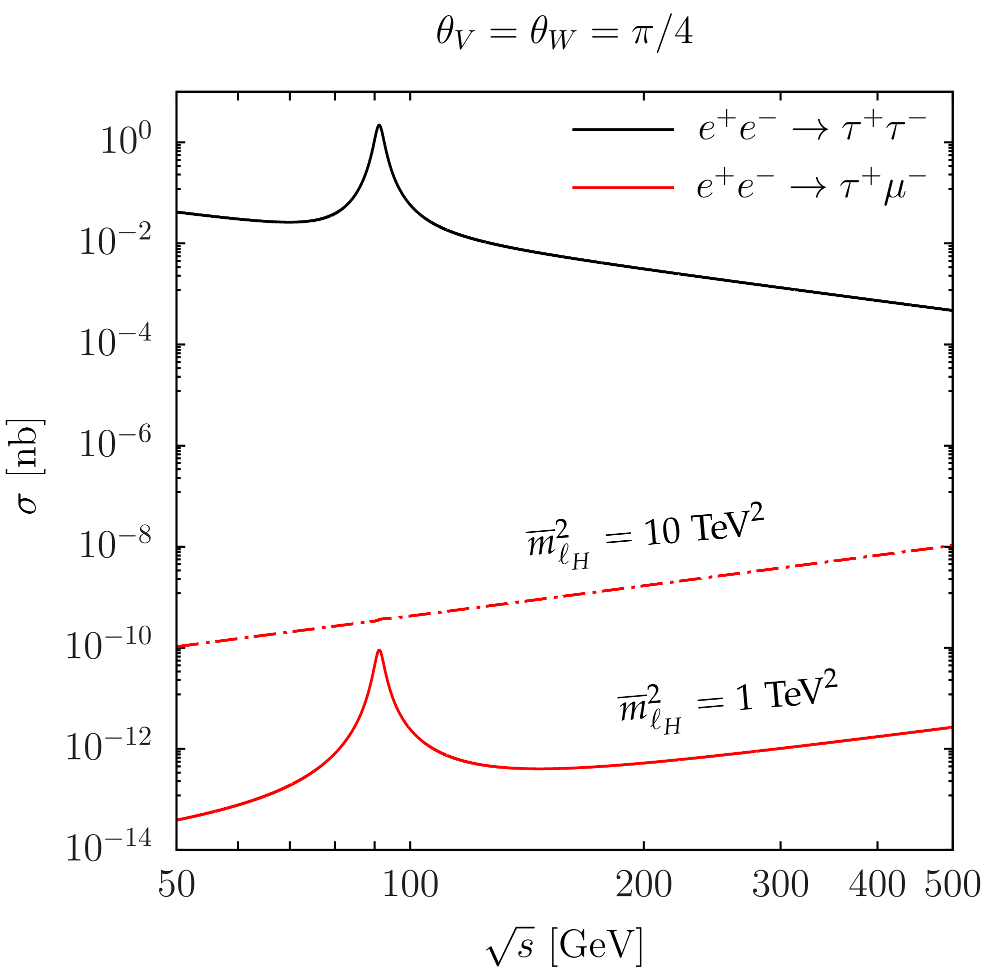
▷ This *tends to happen* **when both l_H and $\tilde{l}^c$ are included**: (as in SUSY)

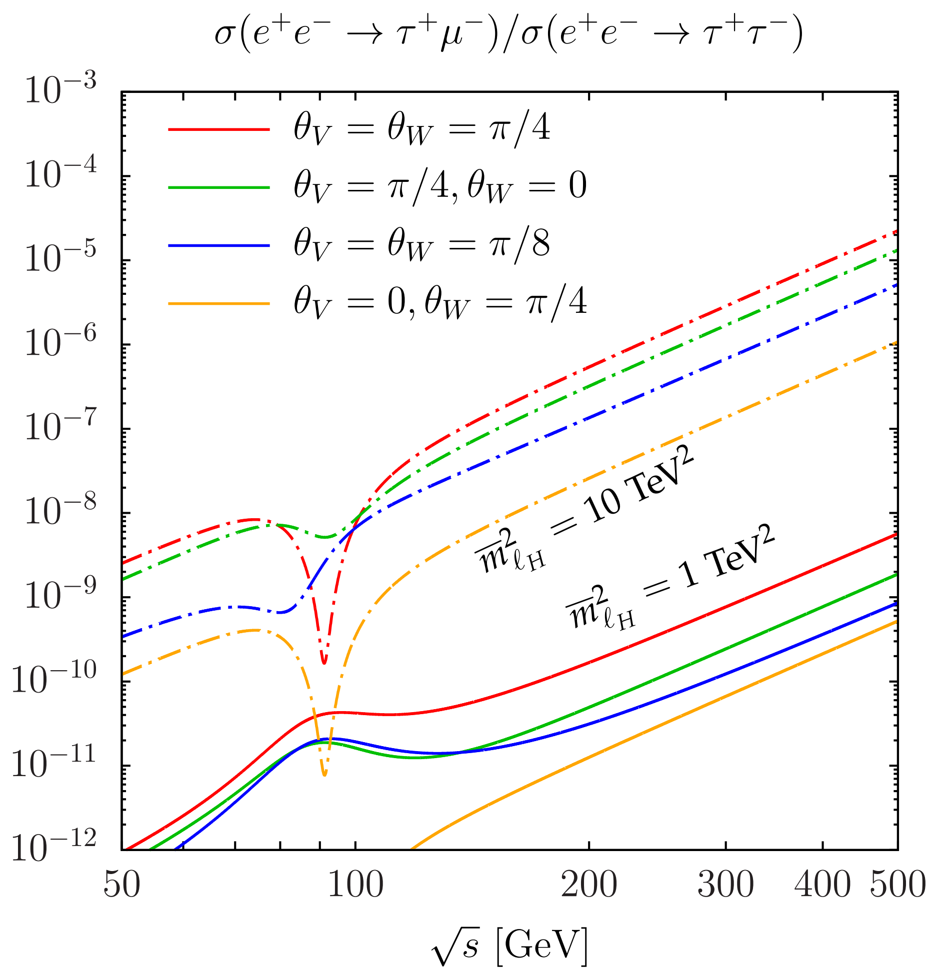
$$\frac{\mathcal{B}(\tau\to\mu e\bar{e})}{\mathcal{B}(\tau\to\mu\mu\bar{\mu})}\approx 5$$

- **When $\tilde{l}^c$ is ignored** (decouples) **the other terms dominate**: (in contrast to SUSY)

$$\frac{\mathcal{B}(\tau\to\mu e\bar{e})}{\mathcal{B}(\tau\to\mu\mu\bar{\mu})}\approx 1$$







▷ At future FCC-ee or ILC ($\int \mathcal{L} dt \sim 10$ ab$^{-1} = 10^{13}$ nb^{-1}) one expects $\sim 10^{10}$ $\tau^+\tau^-$ pairs

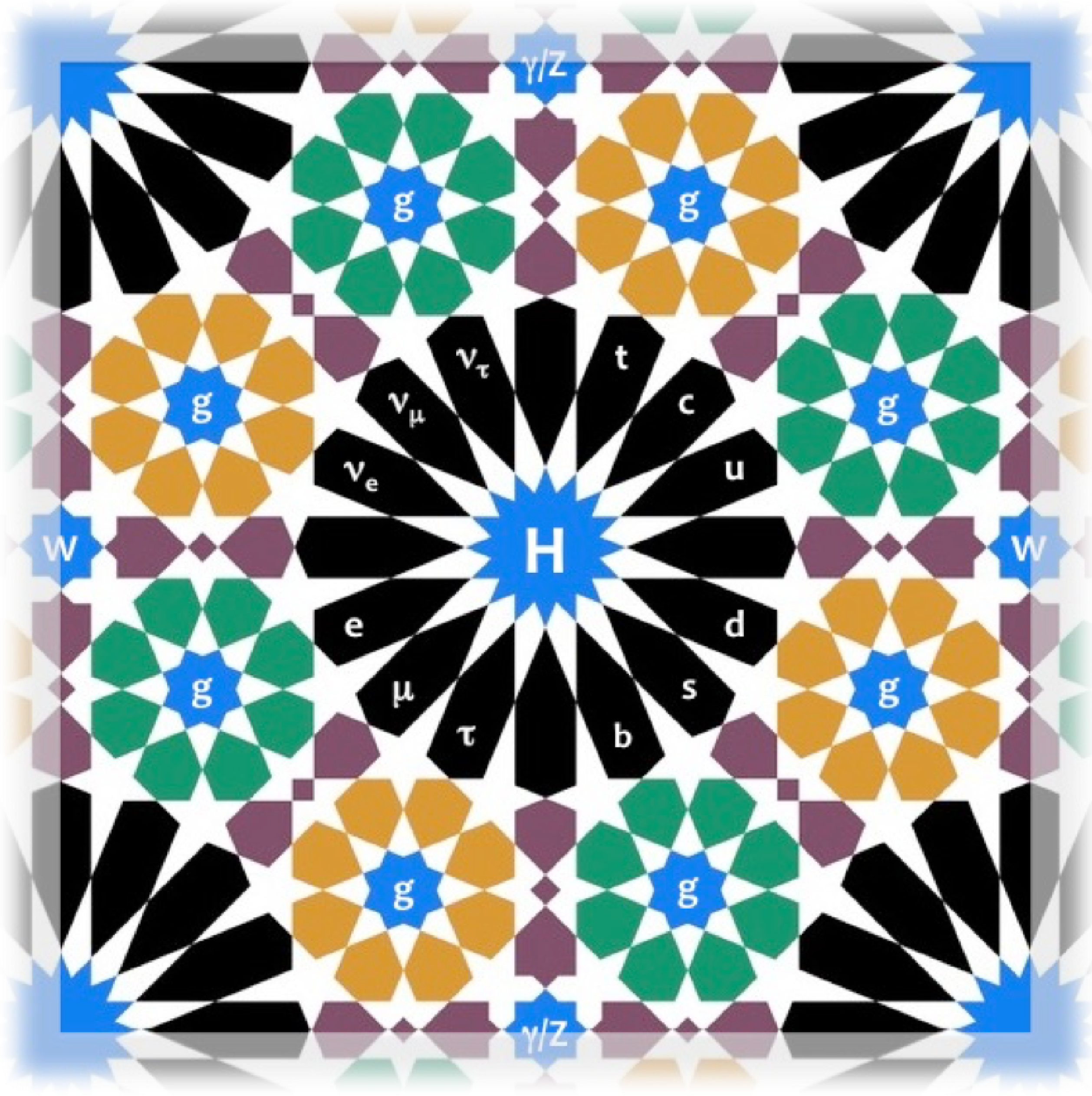
Conclusions

- The one-loop predictions for flavor violating processes in the LHT are UV-finite when *all* Goldstone interactions compatible with gauge and T symmetry and *all* T-odd leptons are included

- LFV is due to misalignment of standard fermions with:
  - ▷ **mirror leptons** (ν_H, ℓ_H): well known source of LFV
  - ▷ **mirror partners** ($\tilde{\nu}^c$, $\tilde{\ell}^c$): new so far ignored, *needed* to make Higgs amplitudes finite

- Mirror partners
  - ▷ do not decouple in $h \to \bar{\ell}_1 \ell_2$, involving couplings to pseudo-Goldstone scalar triplet Φ
  - ▷ decouple in $Z \to \bar{\ell}_1 \ell_2$, $\ell_1 \to \ell_2 \gamma$, $\ell \to \ell_1 \ell_2 \bar{\ell}_3$, μN $\to$ eN, hence may be ignored
  - ▷ BUT, *if* mirror partner masses are of same order as the other T-odd particles, all their contributions have similar size in Higgs and gauge-mediated LFV processes

- **Flavor provides complementary constraints to LH models** particularly for $\mu - e$ transitions: small misalignment ($\sin 2\theta$, $\delta \lesssim 0.01$) *or* heavy scale



Thank You!