

Macroeconomics
Problem Set 1

1. Consider the Infinite Horizon Model as seen in class, where the production function is given by

$$Y_t = AK_t^\alpha L_t^{(1-\alpha)} \quad (1)$$

Show that if factor prices (w_t and R_t) are equal to factor marginal productivities, firm's profits (π_t) are 0.

2. Consider the Infinite Horizon Model as seen in class. Show that from the household's constraint and the firm's optimality conditions, it is possible to obtain the economy feasibility constraint ($Y_t = c_t + i_t$).

3. *A simple Infinite Horizon Model.* Consider an Infinite Horizon Model where households devote their entire unit of time to the labor market ($l_t = L_1 = 1$; no labor hours decision), and where the instantaneous household's utility function is given by

$$U(c_t) = \log c_t \quad (2)$$

Also, the production function is given by

$$Y_t = AK_t^\alpha L_t^{(1-\alpha)} \quad (3)$$

- a) Write the household's maximization problem and obtain its optimality conditions.
b) Write the firm's maximization problem and obtain its optimality conditions
d) Find the steady state values for the endogenous variables assuming that values of the model's parameters are: $A = 1$, $\alpha = 0.48$, $\beta = 0.97$, and $\delta = 0.06$.
4. *Introduction to taxation: consumption taxes* Consider an Infinite Horizon Model where households devote their entire unit of time to the labor market ($l_t = L_1 = 1$; no labor hours decision), and where the instantaneous household's utility function is given by

$$U(c_t) = \log c_t \quad (4)$$

Also, the production function is given by

$$Y_t = AK_t^\alpha L_t^{(1-\alpha)} \quad (5)$$

In addition, there exist a Government that taxes household's consumption with a consumption tax rate τ_c , and it uses all the fiscal revenues to finance some government public consumption G_t .

- a) Write the household's maximization problem and obtain its optimality conditions.
b) Write the government's budget constraint and the good market clearing condition.

- c) Find the steady state values for the endogenous variables assuming that values of the model's parameters are: $\tau_c = 0.2$, $A = 1$, $\alpha = 0.48$, $\beta = 0.97$, and $\delta = 0.06$.
5. *Introduction to taxation: capital income taxes* Consider an Infinite Horizon Model where households devote their entire unit of time to the labor market ($l_t = L_1 = 1$; no labor hours decision), and where the instantaneous household's utility function is given by

$$U(c_t) = \log c_t \tag{6}$$

Also, the production function is given by

$$Y_t = AK_t^\alpha L_t^{(1-\alpha)} \tag{7}$$

In addition, there exist a Government that taxes household's net capital income with a capital income tax rate τ_k , and it uses all the fiscal revenues to finance some government public consumption G_t .

- a) Write the household's maximization problem and obtain its optimality conditions.
- b) Write the government's budget constraint and the good market clearing condition.
- c) Find the steady state values for the endogenous variables assuming that values of the model's parameters are: $\tau_k = 0.2$, $A = 1$, $\alpha = 0.48$, $\beta = 0.97$, and $\delta = 0.06$.