Defined Contribution vs Defined Pension: Reforming the Legal Retirement Age

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Abstract

In this paper we analyze the effects of changing Social Security parameters on the optimal legal retirement age. Two Social Security Systems are studied, with opposite results. When the pension scheme has a defined contribution, a more redistributive system will delay the preferred legal retirement age. On the other hand, when the pension benefit is the defined parameter, the increase in the redistribution level will lower this preferred age.

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I. Introduction

Reforms of Social Security systems are now one of the main issues on the economic policy agenda of most industrialized countries. The main measures proposed to neutralize the future financial problems are raising taxes, cutting benefits or/and delaying the age of retirement. Indeed, this latter reform is one of the policy conclusions of Maintaining Prosperity in an Ageing Society, OECD (1998) "...a direct way to encourage people to work longer would be to raise the pensionable age". This increase would improve the financial future of Social Security since it would imply more income from earnings and shorter retirement periods to be financed.

But delaying the retirement age may not be very popular among people. According to recent surveys, most workers claim to be happy with the current retirement age (see Cremer and Pestieau 2003) which suggests that reforms in the legal retirement age have currently become a very delicate matter for governments. It seems more plausible to implement this reform as part of a package of other measures to solve the financing problems of Social Security. In point of fact, an increase in the legal retirement age would allow milder changes, such as a reduced increase in the contribution rate or a lower decrease in pension benefits.

In this paper, we aim at providing an analysis of the preferred legal retirement age. Using a life-cycle model we calculate the optimal legal retirement age and examine its relationship with the rest of the parameters of Social Security, namely, the contribution rate and pension benefits, via the dependency ratio. We seek to underline the importance of correctly choosing the parameter affected by the dependency ratio in order to make retirement age reform easier. Consequently, we distinguish two Social Security programmes. First, a defined contribution scheme where the contribution rate is constant and pension benefits are affected by the dependency ratio, and secondly, a defined pension scheme where a change in the legal retirement age leads to varying contribution rates and thus the pension is the constant parameter.

On the other hand, as mentioned above, reforms in the legal retirement age should be accompanied by changes in the rest of the parameters of Social Security. For that reason, we analyze how the optimal legal retirement age is affected if the government exogenously alters the defined parameters. In other words, through static comparatives, we study the effects on the preferred legal retirement age derived from changes in the intra-generational and inter-generational redistribution levels of Social Security. The results will be opposite depending on the parameter related to the legal retirement age. So, under a defined pension scheme an increase in the redistribution level of Social Security leads to a lower optimal legal retirement

age. But when we analyze the preferred legal retirement age under a defined contribution scheme, we find the opposite result. The optimal response to an increase in the redistribution level of the system will be to postpone the retirement age.
The term 'legal retirement age' usually refers to the age at which benefits are available. However, since there are also strong incentives to stop working after this standard entitlement age, we consider legal retirement in this model as the age at which workers are obliged to leave the labor force, that is, as a mandatory retirement. Indeed, in some countries there are direct restrictions on work beyond the standard age (in Portugal and Spain entitlement to pension benefits beyond the standard age is conditional on complete withdrawal from work), or, frequently, individuals have to leave their current jobs to receive their pensions (see Blondal and Scarpetta 1998,or Gruber and Wise 1999). So, we can observe that the average retirement age in some OECD countries, such as the United Kingdom, Portugal and Ireland is very close to this standard retirement age.[1]
Many studies have analyzed the relationship between retirement and Social Security. Earlier literature has mainly focussed on the effect of the introduction of a pension system on the individual retirement decision (see among others Sheshinski 1978, Crawford and Lilien 1981, Kahn 1988, or Fabel 1994). There is also more recent literature dealing with the retirement decision in a political economy environment (see Casamatta et al. 2005 or Conde-Ruiz and Galasso 2003 and 2004).
The organization of this paper is as follows: Section 2 presents the theoretical model; Section 3 contains the analysis of the agent's optimal retirement age, and the relation with the rest of the parameters of Social Security; Section 4 presents a numerical example that allows us to highlight some interesting results; and Section 5 contains the concluding remarks and the paper ends with the appendix.

II. The Model

We consider a small open economy in which individuals live exactly T years, of which the first R years represent working life. The population consists of agents belonging to the same generation and distributed between a minimum and a maximum wage level, $[w_{min}, w_{Max}]$.
We assume that individuals have a stationary and temporally independent utility function, which is separable and strictly increasing in consumption and leisure. This is written as

$$(1) \qquad U(c_i^t, l^t) = u(c_i^t) + v(l^t)$$

where c_i^t is the consumption at period t in scheme i. The utility of consumption is twice differentiable with $u' > 0$ and $u'' < 0$ for all c_i^t. Let l^t be the leisure at period t, the utility of leisure being $v(l^t) = 0$, while working, and $v(l^t) = v$, while retired. This utility function is similar to Crawford and Lilien (1981) and Sheshinski (1978). In addition, we assume that the coefficient of relative risk aversion is non-increasing and less than one ($\rho_r(c) < 1$).
Agents plan consumption, savings and retirement to maximize the discounted value of utility subject to their lifetime budget constraint. They are assumed to earn a constant stream of wages per unit of time, w_i, until they retire. Later, from a Social Security programme, they receive a constant stream of pension benefits per unit of time, p, τ being the Social Security contribution rate. The amount of hours spent working cannot be varied as it is institutionally set. Therefore, the only way utility coming from leisure can be modified is by changing the legal retirement age.
We consider a perfectly competitive capital market with free lending and borrowing at a fixed interest rate, which is equal to the subjective discount rate $(\delta = r)$. This assumption together with the separability and concavity of the utility function implies constant consumption in all

[1] If there is a possibility of early access to pension benefits with some adjustment to the value of retirement benefits, the average retirement age is usually found between this age at which pensions can be accessed and the standard retirement age. See Blondal and Scarpetta (1998) or Samwick (1998).

periods, that is, $c_i^t = c_i$ for any t. We also assume a constant birth rate, n, equal to the interest rate, $n = r$.
Thus, the indirect utility function of an individual over her life-cycle can be reduced to

$$(2) \qquad U(c_i;R) = \frac{(1-e^{-rT})}{r}u(c_i) + \frac{(e^{-rR}-e^{-rT})}{r}v$$

where

$$(3) \qquad c_i = \frac{1}{(1-e^{-rT})}((1-e^{-rR})w_i(1-\tau) + (e^{-rR}-e^{-rT})p_i)$$

is the constant consumption per unit of time.
In order to obtain the Social Security budget constraint, let us consider a steady-state situation in which the population is growing at a constant rate, n. As seen in Sheshinski's (1978) model, the steady-state age density function is equal to

$$(4) \qquad f(t) = \frac{n}{1-e^{-nT}}e^{-nt}.$$

The Social Security Programme is financed through the Pay-As-You-Go scheme where pensions of retirees are paid by working people through taxes,

$$(5) \qquad \tau(1-e^{-nR})\int_{w_{min}}^{w_{Max}} wf(w)dw = (e^{-nR}-e^{-nT})\int_{w_{min}}^{w_{Max}} p(w)f(w)dw.$$

So, since the government budget is instantaneously balanced, we can obtain the following relationship between the contribution rate, pension benefits and the legal retirement age

$$(6) \qquad (e^{-nR}-e^{-nT})p_i = (1-e^{-nR})\tau W_i,$$

$W_i = [(1-\alpha)\, w_i + \alpha\, w_i]$ being a linear combination of the mean wage , ϖ, and the individual i's wage, w_i, and $\alpha \in [0,1]$, a redistribution degree. Depending on the level of α, the type of Social Security may range from a totally uniform pension benefits scheme ($\alpha = 0$), usually referred to as Beveridgean, to a totally earnings-related pension benefits scheme ($\alpha = 1$), usually referred to as Bismarckian.[2]
It should also be noted that (6) is time invariant, that is, it does not depend on an arbitrary point in time.
In the next section, we analyze the preferred legal retirement age under the two possible scenarios, a defined pension system and a defined contribution system.

III. Retirement age and redistribution level

Let us now study how the optimal legal retirement age of the individual is affected by changes in the parameters that determine the intra-generational and the inter-generational redistribution level of the Social Security System, and how these effects are different depending on the Social Security system in place. It should be stressed that we study a steady-state economy with different Social Security parameters, that is, we are not concerned with the transition period

[2] See Casamatta et al. (2000) for a classification of several OECD countries depending on the redistribution nature of the Social Security system.

from a steady state with one particular determined parameter to another steady state with a different one.[3]

1 Defined pension scheme

Under this scheme, from (6) we get the following contribution rate:

$$(7) \qquad \tau(R,n,P) = \frac{(e^{-nR} - e^{-nT})}{(1-e^{-nR})}P$$

where $P = p_i/W_i$ gives us the constant ratio between the individual pension and the linear combination of the mean wage and the individual i's wage.[4]
Substituting (7) in (3) the constant consumption per unit of time is as follows

$$(8) \qquad c_i = \frac{(1-e^{-rR})}{(1-e^{-rT})}w_i\left(1 - \frac{(e^{-nR} - e^{-nT})}{(1-e^{-nR})}P\right) + \frac{(e^{-rR} - e^{-rT})}{(1-e^{-rT})}PW_i.$$

The first and second order conditions of the maximization problem are

$$(9) \qquad \frac{\partial U}{\partial R} = \frac{(1-e^{-rT})}{r}u'(c)(\frac{\partial c}{\partial R}) - e^{-rR}v = 0$$

$$(10) \qquad \frac{\partial^2 U}{\partial R^2} = \frac{(1-e^{-rT})}{r}\left(u''(c)(\frac{\partial c}{\partial R})^2 + u'(c)(\frac{\partial^2 c}{\partial R^2})\right) + re^{-rR}v < 0.$$

As in Sheshinski (1978), in order to obtain an interior solution, let us assume that $w_i \geq PW_i$.[5]
Let $R^*_{p,i}$ be the optimal legal retirement age for agent i, that is, the agent with wage w_i. We now analyze the behaviour of the optimal retirement age with respect to the wage. From the F. O. C. and the implicit function theorem, we obtain the following expression

$$(11) \qquad \frac{\partial R^*_{p,i}}{\partial w_i} = -\frac{re^{-rR}[1+P\alpha][u'(c_i)(1-\rho_r(c))] + \eta}{\frac{\partial^2 U}{\partial R^2}}$$

where η is a positive term.[6] So, since (2) is strictly concave with respect to the retirement age, and the relative coefficient of risk aversion is lower than unity and non-increasing, we obtain that $R^*_{p,i}$ is increasing with the wage.
This result coincides with the empirical data, which suggests that lower earners have a higher propensity to retire early, see Blondal and Scarpetta (1998).
Given that there is heterogeneity on wages, there will be different optimal legal retirement ages. So, since $R^*_{p,i}$ is a monotonic function of w_i.,we will assume that the implemented retirement age will be that of the individual with the median wage.
Hence, the next step is to analyze the effects on the preferred legal retirement age of changes in the redistribution parameters. From the F.O.C. and the implicit function theorem, we obtain the following expressions

$$(12) \qquad \frac{\partial R^*_{p,i}}{\partial \alpha} = -\frac{re^{-rR}P\left[u'(c_i)(1-\rho_r(c)) - u''(c_i)w_i\left(1-e^{-rT}\right)\right][w_i - \varpi]}{\frac{\partial^2 U}{\partial R^2}}$$

[3] Our analysis should be interpreted from a long-run perspective.
[4] This constant ratio implies that the changes in the redistribution degree, α, will affect the pension benefits, p_i, instead of affecting the contribution rate, τ. That is, if the redistribution degree increases, and the individual's wage is lower than the mean one, then his pension benefits will increase. On the other hand, for $\alpha = 1$, this ratio P will be the well-known replacement ratio.
[5] This is a plausible assumption implying that the wage earned during the working period is at least as large as the pension received during the retirement period.
[6] The value of η is the following: $\eta = -u''(w_i)re^{-rR}P(1-\alpha)\varpi(1-e^{-rT})$.

$$(13) \qquad \frac{\partial R^{*}_{p,i}}{\partial P} = -\frac{re^{-rR}\alpha\left[u'(c_i)(1-p_r(c)) - u''(c_i)w_i\left(1-e^{-rT}\right)\right][w_i - \varpi]}{\frac{\partial^2 U}{\partial R^2}}.$$

Let $R^{*}_{p,med}$ be the optimal legal retirement age for the agent with the median wage. If $w_{med} < \varpi$, which is the usual case, (12) and (13) will be negative, and therefore, an increase in the level of redistribution (higher α or higher P) would decrease the preferred legal retirement age of the median wage worker, the age that would be implemented. Hence, the next proposition can be formulated as

Proposition 1 Consider a median voter with a wage level $w_{med} < \varpi$. Under a defined pension scheme, increases in the redistribution level of the Social Security System lead to a lowering of the legal retirement age.
Proof: It follows immediately from (12) and (13).

This result can be explained through the income and the substitution effects. In order to do that, we use the constant consumption per unit of time. Given that the interest rate is equal to the birth rate, (8) may be expressed in the following way:

$$(14) \qquad c_i = \frac{1}{(1-e^{-rT})}\left((1-e^{-rR})w_i + \phi_P\right),$$

with

$$(15) \qquad \phi_P = (e^{-rR} - e^{-rT})P\alpha(\varpi - w_i)$$

being the present discounted value of net benefits from Social Security under a defined pension scheme.

For an individual with a wage lower than the mean one, an increase in any redistribution parameter will raise her net benefits from Social Security, and therefore, her income. Since leisure is a normal good, the income effect will lead to an increase in the retirement period. On the other hand, since increases in P or α lead to larger ϕ_P, and $\partial\phi_P/\partial R < 0$, a delay on the retirement age will lead to a lower increase in c_P. That is, an increase in the redistribution level will reduce the relative price of the retirement in terms of consumption, and therefore the agent will substitute consumption for leisure lowering her retirement age.

As both effects have the same sign, either intra- or inter-generational increases in the redistribution level of the Social Security System will give rise to an unambiguous effect on the optimal legal retirement age: they will lower it.

The underlying economic intuition is the following. An increase in any of the two redistribution parameters will also augment the rate of return of the PAYG pension system for those individuals with wages lower than the mean wage (the majority of the population). As a consequence, individuals may have incentives to enlarge the size of the system and to rely more heavily on public savings. This can be achieved by lowering the legal retirement age under the constant pension system, which would increase the contribution rate.

2 Defined contribution scheme

Under this scheme, from (6) we get the following pension benefits:

$$(16) \qquad p_i(R, n, \tau, W_i) = \frac{(1-e^{-nR})}{(e^{-nR} - e^{-nT})}\tau W_i.$$

Substituting (16) in (3) the constant consumption per unit of time is as follows

$$(17) \qquad c_i = \frac{(1-e^{-rR})}{(1-e^{-rT})}w_i(1-\tau) + \frac{(e^{-rR} - e^{-rT})}{(1-e^{-rT})}\left(\frac{(1-e^{-nR})}{(e^{-nR} - e^{-nT})}\tau W_i\right).$$

It is easy to check that (2) is again strictly concave with respect to R. So, now, let $R^*_{\tau,i}$ be the optimal legal retirement age for agent i. Newly, we analyze the behaviour of the optimal retirement age with respect to the wage. From the F.O.C. and the implicit function theorem, we obtain the following expression

$$(18) \qquad \frac{\partial R^*_{\tau,i}}{\partial w_i} = -\frac{re^{-\gamma R}[1-\tau\alpha][u'(c_i)(1-p_r(c))]}{\frac{\partial^2 U}{\partial R^2}}.$$

We obtain that $R^*_{\tau,med}$ is also increasing with the wage.[7] Therefore, we again assume that the implemented retirement age will be that of the individual with the median wage.
Newly, we analyze the effects of the intra- and inter-generational redistribution levels of the Social Security on the preferred legal retirement age. From the F.O.C. and the implicit function theorem, we obtain the following expression

$$(19) \qquad \frac{\partial R^*_{\tau,i}}{\partial \alpha} = -\frac{re^{-\gamma R}\tau(u'(c_i)(1-p_r))(\varpi - w_i)}{\frac{\partial^2 U}{\partial R^2}},$$

$$(20) \qquad \frac{\partial R^*_{\tau,i}}{\partial \tau} = --\frac{re^{-\gamma R}\alpha(u'(c_i)(1-p_r))(\varpi - w_i)}{\frac{\partial^2 U}{\partial R^2}}.$$

Let $R^*_{\tau,med}$ be the optimal legal retirement age for the agent with the median wage. Since $w_{med} < \varpi$., (19) and (20) will be positive, and therefore, an increase in the level of redistribution (higher α or higher τ) would delay the preferred legal retirement age of the median wage worker, the age that would be implemented. Hence, the next proposition can be formulated as

Proposition 2 Consider a median voter with a wage level $w_{med} < \varpi$. Under a defined contribution scheme, increases in the redistribution level of the Social Security System lead to a delay in the legal retirement age.
Proof: It follows immediately from (19) and (20).

Again, we can explain this result through the income and the substitution effects. The constant consumption, (17) may be expressed in the following way:

$$(21) \qquad c_i = \frac{1}{(1-e^{-\gamma T})}\left((1-e^{-\gamma R})w_i + \phi_\tau\right),$$

with

$$(22) \qquad \phi_\tau = (1-e^{-\gamma R})\tau\alpha(\varpi - w_i)$$

being the present discounted value of net benefits from Social Security under a defined contribution scheme.
For an individual with a wage lower than the mean one, an increase in any redistribution parameter will raise her net benefits from Social Security, and therefore, her income. Since leisure is a normal good, the income effect will lead to an increase in the retirement period. On the other hand, since increases in τ or α lead to larger ϕ_τ, and $\partial\phi_\tau/\partial R > 0$, a delay on the retirement age will lead to a larger increase in c_τ. That is, an increase in the redistribution level will raise the relative price of the retirement in terms of consumption, and therefore the agent will substitute leisure for consumption delaying her retirement age.
Since the substitution effect is larger than the income one, intra- or inter-generational increases in the redistribution level of the Social Security System will delay the optimal legal retirement age.

[7] Under this pension system, if the coefficient of relative risk aversion were higher than unity, the results would be the opposite, that is, $R^*_{\tau,med}$ would be decreasing with respect to the wage.

The underlying economic intuition is similar to that mentioned above. Now, in order to enlarge the size of the Social Security system and to rely more heavily on public savings, the majority of the population (those individuals with wages lower than the mean wage) will prefer to delay the legal retirement age as the rate of return of the PAYG pension system is increased due to the larger redistribution parameters, τ or α. This higher retirement age would imply larger pension benefits.

On the other hand, from (15) and (22) we can deduce the following result. If there is no intra-generational redistribution, that is, $\alpha = 0$, the Social Security system does not affect the retirement decision, that is, the preferred legal retirement age will be the same under both systems. But if this parameter grows and the wage of the agent is lower than the mean wage, the present value of postponing retirement will be larger under a defined contribution scheme than under a defined pension scheme. As a consequence, $R^*_{\tau,med}$ would be higher than $R^*_{p,med}$.This result can be stated in the following proposition.

Proposition 3 Let $\alpha > 0$. Let (τ, P) be such that $\tau = (n, P, R^*_{p,med})$, then $R^*_{\tau,med} > R^*_{p,med}$.

Proof:: See appendix.

In addition, we notice that although intra-generational redistribution changes occur only in the retirement period, and the inter-generational ones (P and τ) affect the entire life cycle, from (15) and (22) it is easy to check that with $r = n$ changes in α have the same effect on the optimal retirement age than changes in P or τ. The reason is that the present discounted value of changes in α is equal to the present value of changes in P or τ.

But if we allow the interest rate to be higher than the birth rate, these present values will no longer be equal, and therefore, the effects on the optimal legal retirement age will also be different. This is illustrated in the numerical example in the following section.

IV. Numerical Example

For simplicity, all the previous analysis has been made under the assumption that the interest rate must be equal to the population growth rate. Since the most empirically relevant case is when $r > n$, in this extension we relax the aforementioned assumption.

When the interest rate exceeds the population growth rate, the analytical treatment becomes intractable. The former relationships between Social Security parameters and optimal legal retirement ages under each scheme cannot be mathematically analyzed. Instead, we proceed to calculate the optimal values of the legal retirement age for some alternative levels of α, τ and P. We use the square root function, $u(c) = \sqrt{c}$. The results are summarized in Table 1:

TABLE 1

Optimal legal retirement ages

	Defined pension scheme			Defined contribution scheme		
P	1.126	1.126	1.136	0.486	0.500	0.691
τ	0.2	0.3	0.17	0.2	0.2	0.3
α	0.5	0.6	0.5	0.5	0.6	0.5
$R^*_{j,med}$	45.00	40.10	46.71	32.80	32.95	32.03

Note: $R^*_{j,med}$ for $j = p, \tau$.

The first three rows show different values of α, τ and P . The last row shows the preferred legal retirement ages under each scheme. The first three columns are referred to the defined pension scheme, and the last three columns to the defined contribution scheme.[8] Notice that in the

[8] These optimal retirement ages have been calculated for $r = 0.025$, $n = 0.020$, $T = 60$, $w_{med} = 1$, $\varpi = 1.2$ and $v = 0.5547$. It also has to be noted that for $P = 1.126$, $n = 0.020$, $T = 60$ and $R = 45$, the contribution rate in the defined pension scheme is $\tau = 0.2$.

defined pension scheme, the values of the contribution rate are residually determined from the Social Security budget constraint. On the other hand, in the defined contribution scheme *P* is the parameter residually obtained through the budget constraint.

The second and the fourth columns show the consequences on the preferred legal retirement age of exogenously increasing the intra-generational redistribution degree α. The third and the fifth columns show the effects of exogenously raising the inter-generational redistribution parameters, *P* and τ respectively. It has to be noted that these exogenous changes alter the values of the residually obtained parameters, the contribution rate in the first case and *P* in the second case. Indeed, the lower retirement age obtained in the defined contribution scheme is what explains the smaller amount of *P* in this scheme.

Notice that in this example, with the interest rate being higher than the population growth rate, the different behaviour of the individual depending on the Social Security System is maintained. However, it should be pointed out that now changes in α affect the optimal legal retirement age in a different way that changes P or τ.

We observe that under a defined pension scheme, a change from $\alpha = 0.5$ to $\alpha = 0.6$ reduces the optimal age from $R^*_{p,med} = 45$ to $R^*_{p,med} = 40.10$. However, an increase in the ratio *P* delays the optimal age from $R^*_{p,med} = 45$ to $R^*_{p,med} = 46.71$. The same thing is observed under a defined contribution scheme. A change from $\alpha = 0.5$ to $\alpha = 0.6$ delays the optimal age from $R^*_{\tau,med} = 32.80$ to $R^*_{\tau,med} = 32.95$. However, an increase in the contribution rate τ causes a reduction from $R^*_{\tau,med} = 32.80$ to $R^*_{\tau,med} = 32.03$.

The reason for this different behaviour is that changes in the intra-generational redistribution parameter α affect only the present value of pension benefits, but changes in the inter-generational ones, *P* and τ, it will affect the net present value of benefits from less payments to Social Security. If we allow the population growth rate be lower than the interest rate, net benefits from Social Security, (15) and (22), can be rewritten in the following way:

$$(23) \qquad \phi_P = P\left((e^{-rR} - e^{-rT})W_i - (1 - e^{-rR})\frac{(e^{-nR} - e^{-nT})}{(1 - e^{-nR})}w_i\right)$$

and

$$(24) \qquad \phi_\tau = \tau\left((e^{-rR} - e^{-rT})\frac{(1 - e^{-nR})}{(e^{-nR} - e^{-nT})}W_i - (1 - e^{-rR})w_i\right).$$

If the interest rate is higher than the birth rate, ϕ_i may be negative even with $w_i < \varpi$. If this were the case, as in our numerical example, an increase in *P* or τ would decrease the rate of return of the PAYG pension system. As a consequence, individuals may have incentives to limit the size of the system and to rely more heavily on private savings. This can be achieved by delaying the legal retirement age under the constant pension system, which would reduce the contribution rate, and lowering it under the constant contribution system, which would reduce the pension benefits.

But if the modified parameter were the intra-generational one, then, only the present value of pension benefits would be altered. And, since $w_i < \varpi$, an increase in α would augment the rate of return of the PAYG pension system. Unlike the previous case, individuals now have incentives to increase the size of the system and to rely more heavily on public savings. This can be achieved by reducing the legal retirement age under the constant pension system, which would raise the contribution rate, and delaying it under the constant contribution system, which would increase the pension benefits.

V. Conclusions

There is a current debate in most OECD countries as to the reforms that must be adopted in order to solve the financing problems of Social Security Systems. The two main possibilities, increasing the contribution tax rate or a reduction in the pension benefits, might go together with a delay in the pensionable retirement age, which would soften either the contribution rate increase or the pension benefits reduction. For that reason we have analyzed the effect on the preferred legal retirement age derived from a change in the redistribution parameters of Social Security. Given that we have proved that optimal retirement ages increase with wages, we have considered the median wage worker's optimal choice as the implemented legal retirement age. We have also assumed that the median wage is lower than the mean wage, the usual case in most industrialized countries.

Effects on the optimal legal retirement age of redistributive changes will depend on the relationship between the birth and the interest rates. When they are equal (or very similar), present values of varying the inter-generational parameter and of varying the intra-generational one are (almost) equivalent and therefore, effects derived from these changes will also be equivalent. In such cases, increases in the redistribution level of the Social Security system, (both intra- and inter-generational) would lead to a delay in the preferred legal retirement age under a defined contribution system, but the same increase would provoke the opposite effect under a defined pension scheme, lowering the legal retirement age.

On the other hand, if the interest rate is higher enough than the birth rate, present values of varying the inter-generational parameter and of varying the intra-generational one are no longer equivalent, and consequently, effects derived from these changes on the optimal retirement age might also be different. We have proved this with a numerical example where, under a defined pension, the legal retirement age is delayed if the redistribution from high-wage to low-wage workers is increased but it is advanced if the altered parameter is the inter-generational one. However, under a defined contribution scheme, we get the opposite results.

These results suggest that governments aiming to delay the legal retirement age in order to soften possible changes in Social Security parameters should have to take into account which parameter has to be affected by the dependency ratio in order to involve a lower degree of compulsion in the reform of the legal retirement age.

Appendix: Proof Proposition 3

We have to prove that if $\tau(n, p, R^*_{p,med}) = \tau$, then $R^*_{\tau,med} > R^*_{p,med}$ with $w_i < \varpi$. Let us recall the constant consumption per unit of time under each scheme

$$(A1) \qquad c_p = \frac{1}{(1-e^{-rT})}\left((1-e^{-rR})w_i + (e^{-rR}-e^{-rT})P\alpha(\varpi - w_i)\right)$$

and

$$(A2) \qquad c_\tau = \frac{1}{(1-e^{-rT})}\left((1-e^{-rR})w_i + (1-e^{-rR})\tau\alpha(\varpi - w_i)\right).$$

Given that for $R = R^*_{p,med}$, if $\tau(n, p, R^*_{p,med}) = \tau$, then c_p will be equal to c_τ, we have to prove that at that point

$$(A3) \qquad \frac{\partial c_\tau}{\partial R} > \frac{\partial c_p}{\partial R}.$$

If this inequality holds, the single-peakness property of the utility function implies that $R^*_{\tau,med} > R^*_{p,med}$. We calculate $\partial c_p/\partial R$ and $\partial c_\tau/\partial R$, in order to compare them:

$$(A4) \qquad \frac{\partial c_p}{\partial R} = ne^{-nR}(w_i - P\alpha(\varpi - w_i))$$

$$(A5) \qquad \frac{\partial c_\tau}{\partial R} = ne^{-nR}(w_i + \tau\alpha(\varpi - w_i)).$$

Given that $w_i < \varpi$, we obtain that (A3) holds, and therefore $R^*_{\tau,med} > R^*_{p,med}$.

Q.E.D.

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