Improved estimates for the heat equation on exterior domains in $\mathbb{R}^d$

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Abstract

We give large-time uniform asymptotic estimates for the Dirichlet heat kernel in the complement Ω of a compact set with certain technical regularity assumptions. In particular, in dimension 3 and higher we show that it approaches a given multiple of $\phi\Gamma$, where Γ is the usual full-space fundamental solution and ϕ is the unique positive solution to $\Delta\phi = 0$ on Ω with $\phi = 0$ on $\partial\Omega$. In dimensions 1 and 2 we also give specific asymptotic estimates which are more involved to state. All estimates include an explicit rate of approach to the asymptotic profiles, and are also given for general solutions of the heat equation on Ω.

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